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Inheritance taxation in a model with intergenerational time transfers

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Abstract

We consider a two-period overlapping generation model with rational altruism *à la* Barro, where time transfers and bequests are available to parents. Starting from a steady state where public spending is financed through taxation on capital income and labor income, we analyze a tax reform that consists in a shift of the tax burden from capital income tax towards inheritance tax. In the standard Barro model with no time transfer and inelastic labor supply, such a policy decreases steady-state welfare. In our setting, inheritance tax modifies parents' trade-off between time transfers and bequests. We identify situations where the tax reform increases welfare for all generations. Welfare improvement mainly depends on the magnitude of the effect of higher time transfers on the labor supply of the young.

Keywords: family transfers, altruism, time transfers, inheritance tax.
JEL Classifications: D64, H22, H24, J22.

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1 Introduction

Inheritance taxation is often connected with the hampering effects on savings and capital accumulation, in the same way as capital income taxation. However, both taxes have a different impact on labor supply. An approach that has been followed in the literature (see e.g. Kopczuk, 2013 or Kindermann et al., 2018) emphasizes that inheritance taxation by reducing wealth transmission within the family encourages working age people to work more. We highlight another mechanism that passes through family time transfers. Indeed, estimates of time transfers give them as much importance as cash transfers making them likely to affect the labor supply of the working age people. By taking time transfers into account, a trade-off is introduced between both kinds of transfers. Inheritance taxation then acts as a subsidy on time transfers, having a positive effect on the labor supply of the working age people, and introducing a mechanism that does not appear with capital income taxation.

In this paper, we show that replacing a capital income tax with an inheritance tax can be Pareto-improving. We consider a second-best situation where the government has been forced to introduce distorting taxes and in which the taxation of capital income is higher than its first-best level. The tax reform consists in reducing capital income tax and replacing it with an inheritance tax. The fall in capital income taxation may compensate the negative effect of inheritance taxation on savings and capital accumulation.[1] Indeed, the tax reform can be designed in order to keep the capital-labor ratio constant, using the notion of balanced growth path incidence introduced by Stiglitz (1978).

If the tax reform keeps the capital-labor ratio constant, higher labor supply can then enhance the resources produced by the private sector. The positive effect that inheritance taxation can introduce is based on the idea that the working age people devote a significant part of their time to domestic production in which their parents' contribution in the form of time transfers can be useful (through childcare, clean up, meals, gardening and so on). Unlike capital income taxation, inheritance tax directly affects the trade-off between cash transfers and time transfers. It provides an incentive for grandparents to participate in the domestic tasks of their children and thus impacts their labor supply.

A number of empirical studies suggest that time transfers from parents to their children are substantial and on average almost as important as monetary transfers. Cardia and Ng (2003, Table 1) uses the Health and Retirement Study (HRS) of 1992 and report that the mean time transfer for total sample (7547 households) of 325 hours has a value of \$1950 (using a time cost of \$6 per hour), which is similar to the sample mean of \$1868 for monetary transfers. Studies based on the Survey of Health, Ageing and Retirement in Europe (SHARE, survey conducted since 2004 in ten Western European countries) show that parents' time transfers to children consist

[1] For instance, Michel and Pestieau (2005) show that inheritance tax may not achieve the objective of equalizing the distribution of income because of capital accumulation effects.

mainly in childcare (see Attias-Donfut et al., 2005 or also Albertini et al., 2007). According to Wolff and Attias-Donfut (2007), two-fifths of grandparents keep their grandchildren every week. A common finding is that grandparents still support parents' home production with household tasks for instance. Ho (2015) uses data from the Health and Retirement Study (HRS) and gets that time transfers from grandparents are also substantial for the US households. He focuses on childcare and obtain that the main source of daycare for children below five with employed mothers is provided by grandparents. Moreover, empirical studies confirm that downward transfers in time and money dominate upward transfers. For instance, Schoeni (1997) using the 1988 Panel Study of Income Dynamics (PSID) analyzes time and money transfers given and received by US individuals according to age and obtain that young adults receive more time and money transfers than people of other ages.

In the following, we consider a two-period overlapping generation model with rational altruism *à la* Barro (1974), meaning that parents care about the welfare of their children. As is well known, in the standard model of Barro (1974) without time transfers, both taxes on inheritance and on capital income reduce the steady-state capital-labor ratio, moving the economy away from the Golden rule of capital accumulation. We show that implementing a positive inheritance tax rate reduces households' welfare in the long run even if the tax reform consists in a shift from capital income tax towards inheritance tax designed in such a way that the steady-state capital-labor ratio remains constant.

To take account of labor supply effect of the tax reform, we consider a model closed to the one developed by Cardia and Ng (2003) and Cardia and Michel (2004).[2] In each period of life, households consume a composite good that aggregates market good and home production. Labor supply decision depends on the trade-off between formal work and home production. Indeed, taking into account time transfers allows to analyze the effect of the inheritance tax on the time spent on home production by the retirees. If the latter effect is negative, time endowment of the next generation increases, allowing for higher income from formal work. Nevertheless, the positive effect on labor supply has to be balanced with the potential reduction in private wealth received from the parent.[3]

Assuming that the tax reform is designed in order to leave the steady-state capital-labor ratio constant, we identify situations which are welfare-improving. First, households' life-cycle utility in

[2] Belan et al. (2010) have also used this framework in order to analyze the effect of retirement age on labor supply.

[3] Most studies find a partial substitution between money and time transfers (Schoeni, 1997, Cardia and Ng, 2003, or Attias-Donfut et al., 2005). In our model, this substitution depends crucially on the substitutability between time and market good in home production of the old and of the young adult. Cardia and Ng (2003) who focus on grandparent childcare consider elasticity of substitution larger than, but not far from, unity. They back up their assumption on the fact that time spent by mother for childcare tends to increase with their education level, leaving room for a low substitution between time and other inputs in this activity (Leibowitz, 1974). More generally, the mesure of substitutability between market good and time for home production has received attention in the literature. Unity is generally considered as a reasonable lower bound. The different studies report value in the range of 1.4 to 2.5 (see e.g. Rogerson and Wallenius, 2016, Aguiar and Hurst, 2007, Chang and Schorfheide, 2003, McGrattan et al., 1997). Consequently, all these estimates clearly give room for substitutability between money and time transfers.

steady state is likely to increase when substitutability in retirees' home production between time and consumption of market good is high. Indeed, the substitutability enhances the positive effect of the reform on labor supply since time transfers react more strongly to the reform. This results in increased disposable resources in market good and leads to higher market good consumption in both periods. Secondly, even if the substitutability is small, the tax reform may have a positive effect on utility through the size of the additional production of market goods generated by the increase in labor supply. We show that the strength of the latter effect depends crucially on the gap between the marginal rate of transformation and the marginal rate of substitution between consumption in market goods and formal work. The reform is likely to increase utility if an additional unit of labor allows the production sector to generate more market goods than necessary for leaving individual with the same level of utility. Using a numerical illustration, we analyze the effect of the tax reform on welfare along the transitional dynamics, and identify situations where the tax reform increases the welfare of all generations, including the first old.

The paper is organized as follows. In Section 2, the model is presented. Section 3 analyzes the steady-state equilibrium with operative bequests and positive time transfers. Then, in Section 4, we present the tax reform and study its effects on households' utility in steady state. In Section 5, we conduct a numerical illustration in order to study the impact of the tax reform on the whole transitional dynamics. The final section concludes.

2 The model

We consider an overlapping generation model. Time is discrete. Population consists in one representative dynasty where the household of generation t lives for two periods and has one child, born in $t+1$. We consider rational altruism *à la* Barro (1974) from parents to children.

2.1 Households

The household of generation t works during his first period of life (*i.e.* when young, or equivalently parent) and then retires (*i.e.* when old, or equivalently grandparent). Labor supply when young is elastic and depends on the allocation of a unit-time endowment between formal work and home production. In both periods, the household consumes a composite good that aggregates market good and home production. Life-cycle utility writes

$$u\left(f^y\left(c_t^y, T_t^y\right)\right) + v\left(f^o\left(c_{t+1}^o, T_{t+1}^o\right)\right)$$

where u and v are increasing and strictly concave. Function $f^y\left(c_t^y, T_t^y\right)$, respectively $f^o\left(c_{t+1}^o, T_{t+1}^o\right)$, is the quantity of composite good when young, resp. when old. The former is obtained with market good expenditures c_t^y and time devoted to home production T_t^y. In the latter, c_{t+1}^o represents market good expenditures when old, while T_{t+1}^o is time spent in home production. Home production functions f^y and f^o are assumed to be linear homogeneous and concave. Marginal products are strictly positive and strictly decreasing.[4]

Let ℓ_t denote labor supply of the young in the formal sector. Household's decision for labor supply results from the trade-off between formal work and domestic production. Time devoted to home production when young, T_t^y, aggregates time he/she spends in home production $1-\ell_t$ and time transfer received from his/her parent (denoted by λ_t):

$$T_t^y = 1 - \ell_t + \mu\lambda_t \tag{1}$$

where $\mu > 0$ represents the relative efficiency of time transfer.[5] Since the parent is retired, time spent in home production when old is the fraction of the unit-time endowment which is not transferred to his offspring:

$$T_t^o = 1 - \lambda_t \tag{2}$$

In the following, τ_t^w, τ_t^x and τ_t^R are the respective period-t tax rates on wages, bequests and capital income. R_t and w_t denote the gross interest rate and the wage rate. When young, a household born in t receives after-tax wage income $(1-\tau_t^w)\, w_t\ell_t$ and after-tax bequest $(1-\tau_t^x)\, x_t$.[6] These resources are allocated between consumption c_t^y and saving s_t:

$$c_t^y + s_t = (1-\tau_t^w)\, w_t\ell_t + (1-\tau_t^x)\, x_t \tag{3}$$

When old, the household receives a pension b_{t+1} and after-tax capital income $\left(1-\tau_{t+1}^R\right) R_{t+1}s_t$. This income is allocated between consumption c_{t+1}^o and bequest x_{t+1}:

$$c_{t+1}^o + x_{t+1} = \left(1-\tau_{t+1}^R\right) R_{t+1}s_t + b_{t+1} \tag{4}$$

Following Barro (1974), rational altruism means that households enjoy utility of their children. Utility of the household born in t, U_t, depends on consumptions in composite goods in both periods and utility of its offspring U_{t+1}:

$$U_t = u\left(f^y\left(c_t^y, T_t^y\right)\right) + v\left(f^o\left(c_{t+1}^o, T_{t+1}^o\right)\right) + \beta U_{t+1}$$

[4] This formulation is equivalent to the one considered by Cardia and Michel (2004). They assume that households derive their utility in both periods of life ($j = y, o$) from home produced good $q^j = Q^j\left(m^j, T^j\right)$, and market good $c^j - m^j$ where m^j is market good units used in home production. The composite good consumed corresponds then to an aggregation of market and home produced goods: $\bar{f}^j\left(c^j - m^j, q^j\right)$. Our function f^j can be derived through the following maximization: $f^j\left(c^j, T^j\right) = \max_{m^j} \bar{f}^j\left(c^j - m^j, Q^j\left(m^j, T^j\right)\right)$ with the same properties as mentioned in the text provided $\bar{f}^j$ and Q^j are increasing in all their arguments, concave, and linear homogeneous.

[5] The parameter μ represents the capacity of the old parent to help his child. Its level depends for instance on health and geographical distance.

[6] Bequest appears as an *inter vivos* transfer since it is a resource for children in their first period of life, before the death of their parent. As long as children do not face with borrowing constraint, a formulation where they receive the bequest in their second period of life would not change neither the dynamics of capital accumulation, nor consumption bundles and time allocation of all generations.

where β denotes the degree of altruism, $0 < \beta < 1$.

Using equations (1)-(4), both consumptions in market good rewrite

$$c_t^y = (1 - \tau_t^w) w_t [1 - T_t^y + \mu (1 - T_t^o)] + (1 - \tau_t^x) x_t - s_t \tag{5}$$

$$c_{t+1}^o = \left(1 - \tau_{t+1}^R\right) R_{t+1} s_t + b_{t+1} - x_{t+1} \tag{6}$$

The problem of the dynasty at time zero is to maximizes

$$W = \frac{1}{\beta} v\left(f^o\left(c_0^o, T_0^o\right)\right) + \sum_{t=0}^{+\infty} \beta^t \left[u\left(f^y\left(c_t^y, T_t^y\right)\right) + v\left(f^o\left(c_{t+1}^o, T_{t+1}^o\right)\right)\right]$$

subject to (5) for $t \geq 0$, (6) for $t \geq -1$ and

$$\mu (1 - T_t^o) \leq T_t^y \leq 1 + \mu (1 - T_t^o), \quad x_t \geq 0, \quad 0 \leq T_t^o \leq 1, \quad \text{for } t \geq 0,$$

given s_{-1}.[7] Plugging (5)-(6) into W gives household's utility as a function of $(s_t, T_t^y, x_t, T_t^o)_{t \geq 0}$. For an interior solution, this leads to the following first-order conditions:

- with respect to s_t

$$-u_t' f_{c_t^y}^y + \left(1 - \tau_{t+1}^R\right) R_{t+1} v_{t+1}' f_{c_{t+1}^o}^o = 0 \tag{7}$$

  where u_t', $f_{c_t^y}^y$, v_{t+1}' and $f_{c_{t+1}^o}^o$ respectively stand for the partial derivatives $\frac{\partial u_t}{\partial f_t^y}$, $\frac{\partial f_t^y}{\partial c_t^y}$, $\frac{\partial v_{t+1}}{\partial f_{t+1}^o}$ and $\frac{\partial f_{t+1}^o}{\partial c_{t+1}^o}$.

- with respect to T_t^y

$$-(1 - \tau_t^w) w_t f_{c_t^y}^y + f_{T_t^y}^y = 0, \text{ if } \mu (1 - T_t^o) < T_t^y < 1 + \mu (1 - T_t^o) \tag{8}$$

  where $f_{T_t^y}^y$ stands for $\frac{\partial f_t^y}{\partial T_t^y}$.

- with respect to x_t

$$-v_t' f_{c_t^o}^o + \beta (1 - \tau_t^x) u_t' f_{c_t^y}^y = 0, \text{ if } x_t > 0 \tag{9}$$

- with respect to T_t^o

$$v_t' f_{T_t^o}^o - \beta \mu (1 - \tau_t^w) w_t u_t' f_{c_t^y}^y = 0, \text{ if } 0 < T_t^o < 1 \tag{10}$$

  where $f_{T_t^o}^o$ stands for $\frac{\partial f_t^o}{\partial T_t^o}$.

Conditions for an interior solution are not necessarily satisfied at equilibrium. The less critical

[7] The parent moves first and commits to a bequest and a time transfer. We do not consider strategic interactions between generations.

one is the constraint $T_t^y < 1 + \mu(1 - T_t^o)$, which is equivalent to positive labor supply ($\ell_t > 0$). Assuming that it is satisfied remains to consider equilibria where the production sector uses labor. The constraint $T_t^o \geq 0$ should be satisfied if time cannot be fully replaced with market good in home production. Finally, non-negativity constraints on bequests and time transfers deserve some discussion, as well as the the fact that labor supply of the young cannot exceed one: $T_t^y \geq \mu(1 - T_t^o)$. They depend on the utility gains that children obtain from both kinds of transfers. We will go back to this issue after the presentation of the steady-state in the next section.

2.2 Equilibrium

The production sector consists in a representative firm that behaves competitively, and produces output with labor and capital. The production function $F(k, \ell)$ is linear homogeneous and concave, and includes capital depreciation. Marginal products are strictly positive and strictly decreasing. Profit maximization leads to the standard equality between factor prices and marginal products

$$\begin{aligned} w_t &= F_L(k_t, \ell_t) \\ R_t &= F_K(k_t, \ell_t) \end{aligned}$$

where k_t is capital stock. F_L and F_K stand for the partial derivatives of F with respect to labor and capital.

At equilibrium, household savings s_t split into private capital that will be used in $t+1$ and public debt Δ_t

$$k_{t+1} + \Delta_t = s_t$$

In each period, government spending includes pension b_t and a constant per-young amount g that corresponds to acquisition of goods. Government resources come from taxation on labor income, capital income and bequests. Then, public debt accumulates according to the following law of motion:

$$\Delta_t = R_t \Delta_{t-1} + g + b_t - \tau_t^w \ell_t w_t - \tau_t^R R_t s_{t-1} - \tau_t^x x_t \tag{11}$$

where the initial public debt $\Delta_{-1} = \bar{\Delta}_{-1}$ is given. Finally, the resource constraint in period t writes

$$c_t^y + c_t^o + k_{t+1} + g = F(k_t, \ell_t) \tag{12}$$

3 Steady state with positive transfers

We consider steady state where both family transfers are positive. Tax rates, pension and public debt are assumed to be constant over time. From the marginal conditions (7) and (9), the gross interest rate is equal to R_M defined as

$$\beta\left(1-\tau^{x}\right)\left(1-\tau^{R}\right)R_M=1 \tag{13}$$

which characterizes the capital-labor ratio $k/\ell = z_M$ and the wage rate $w_M = F_L\left(z_M, 1\right)$.

From equations (7), (8) and (10), the other marginal conditions of the household problem can be rewritten as equalities between marginal rates of substitution and relative prices:

$$\frac{v' f^o_{c^o}}{u' f^y_{c^y}} = \beta\left(1-\tau^{x}\right) \equiv P^R \tag{14}$$

$$\frac{f^y_{T^y}}{f^y_{c^y}} = \left(1-\tau^{w}\right) w_M \equiv P^y \tag{15}$$

$$\frac{f^o_{T^o}}{f^o_{c^o}} = \mu\frac{\left(1-\tau^{w}\right) w_M}{1-\tau^{x}} \equiv P^o \tag{16}$$

where P^R is the relative price between market good consumption when old c^o and market good consumption when young c^y, and P^y (resp. P^o) is the relative price between time devoted to home production and market good consumption when young (resp. when old). For the young, the relative price P^y corresponds to the net wage.

Time constraint when young (1) gives the household's labor supply

$$\ell = 1 - T^y + \mu\left(1-T^o\right). \tag{17}$$

Then, the resource constraint (12) becomes

$$c^y + c^o + g = C_M\left[1 - T^y + \mu\left(1-T^o\right)\right] \tag{18}$$

where C_M denotes aggregate consumption (including government consumption) per labor unit

$$C_M \equiv F\left(z_M, 1\right) - z_M$$

Consequently, for given tax rates $\left(\tau^w, \tau^R, \tau^x\right)$, relative prices P^y, P^o, P^R and aggregate consumption per labor unit C_M can be computed. Then marginal conditions (14)-(16) and the resource constraint (18) characterize household's choice for consumptions in market good, c^y and c^o, and times devoted to home production, T^y and T^o. Finally, bequests and public debt are respectively obtained from the household's intertemporal budget constraint and the government budget constraint.

Indeed, the household's intertemporal budget constraint (obtained by eliminating s_t from equations (5) and (6)) allows to compute steady-state bequest. Using the time constraint (17) and the relation between relative prices $P^R P^o = \beta\mu P^y$ (deduced from their definitions in (14)-(16)), the intertemporal budget constraint rewrites

$$c^y + P^y T^y + P^R \left(c^o + \beta^{-1} P^o T^o\right) = P^y (1 + \mu) + P^R b + (1 - \tau^x)(1 - \beta) x \tag{19}$$

Bequests are positive if the present value of market good consumption $c^y + P^R c^o$ is higher than the sum of net wage income $(1 - \tau^w) w_M \ell$ and the present value of the pension $P^R b$. As shown by Weil (1987), in the standard Barro framework without time transfers, steady-state bequests are positive, if the long-run capital-labor ratio in the corresponding Diamond economy is below the modified Golden rule.[8] With time transfers, assuming logarithmic utility and Cobb-Douglas technology, Cardia and Michel (2004) have given conditions for the existence of intertemporal equilibria where both transfers are positive. In this particular case, steady-state bequests are positive if the degree of altruism is higher than a threshold which is the same as the one obtained in a model without time transfers, *i.e.* the same as in Weil (1987). Additionally, positive time transfers are obtained for sufficiently high relative efficiency of parents' time transfer μ. But μ should not be too high to avoid situations where the young prefer to supply his/her unit time in the formal sector and rely on parents for domestic production. These results still hold in our model. In Appendix (Section 7), we give conditions for both transfers to be positive and labor supply of the young to be lower than one, assuming logarithmic utility function u and v, and CES home production function f^y and f^o. In the following, since one of our main concerns is the effect of inheritance tax on labor supply of the young through time transfers, we focus on steady-state equilibria where both transfers are positive and labor supply of the young has not reached its upper bound.

4 Fiscal reform

In the following, we assume that the government cannot reach a first-best allocation, that is the government cannot set the tax rates (τ^x, τ^w and τ^R) at zero, and use the second-period transfer b as a lump-sum tax in order to finance government purchases. We assume that, before the tax reform, capital income is taxed at a positive rate $\bar{\tau}^R > 0$, while bequests are untaxed ($\bar{\tau}^x = 0$) and labor income is taxed at a given rate $\bar{\tau}^w \geq 0$. Positive capital income tax rate distorts household's saving decision, leading to a lower capital-labor ratio than the one obtained at a first-best optimum.

The issue we address is whether a tax shift from capital income tax towards inheritance tax would be welfare-enhancing. The tax reform consists to set up a positive inheritance tax rate $\tau^x > 0$ and reduces the capital income tax rate τ^R.

In the following, we conduct the analysis by first assuming that the fiscal reform is designed in order to keep the capital-labor ratio constant at steady state. Therefore, the shift from capital

[8] Weil (1987) assumes uniqueness of the steady-state capital-labor ratio of the corresponding Diamond economy, that is the economy without bequest motive as introduced by Diamond (1965). With multiple steady states, the condition for positive bequest has been derived by Thibault (2000).

income to inheritance tax is such that

$$\left(1-\tau^{x}\right)\left(1-\tau^{R}\right)=1-\bar{\tau}^{R}$$

The tax reform may introduce an intergenerational redistribution of resources. In this section, we focus on the effect of the reform on steady-state life-cycle utility:

$$V=u\left(f^{y}\left(c^{y},T^{y}\right)\right)+v\left(f^{o}\left(c^{o},T^{o}\right)\right) \tag{20}$$

and postpone the issue of intergenerational redistribution to the next section, through a numerical illustration.

The rest of the section decomposes the effects of the tax reform in different settings to better interpret the overall impact. We start from the standard model of Barro (1974) with inelastic labor supply and no time transfer. We show that the tax reform decreases the steady-state household's life-cycle utility even considering a constant capital-labor ratio. We then extend the discussion to elastic labor supply (still without time transfer) in order to analyze the tax reform effect on the young's labor supply and the resulting impact on his disposable resources. The reform increases labor supply, and may improve the household's utility through the rise in the resources for market good consumption. Finally, we consider the complete framework with elastic labor supply and family time transfers in order to take account of the effect of the tax reform on the trade-off between both private intergenerational transfers, which then affect the young's labor supply.

4.1 Tax reform without time transfer at steady state

Consider an economy without time transfer, assuming, for instance, a zero relative efficiency of time transfers ($\mu=0$). At a steady-state equilibrium with positive bequests and zero time transfer, *i.e.* $x>0$ and $T^{o}=1$, the gross interest rate satisfies (13) and determines the capital-labor ratio and the wage rate. Market good consumptions (c^{y} and c^{o}) and time spent to home production when young T^{y} are characterized by marginal conditions (14)-(15) and the resource constraint (18) and vary with the tax reform. Thus, the marginal change in τ^{x} reduces the relative price P^{R} and then modifies the household's intertemporal allocation of resources between market good consumptions in both periods. The magnitude of the effect crucially depends on the elasticity of substitution between the composite goods f^{y} and f^{o}. Let us denote by σ^{u}, the absolute value of this intertemporal elasticity of substitution. Then

$$\frac{df^{y}}{f^{y}}-\frac{df^{o}}{f^{o}}=\sigma^{u}\frac{d\left(\frac{f^{y}_{c^{y}}P^{R}}{f^{o}_{c^{o}}}\right)}{\frac{f^{y}_{c^{y}}P^{R}}{f^{o}_{c^{o}}}}=\sigma^{u}\left(\frac{df^{y}_{c^{y}}}{f^{y}_{c^{y}}}-\frac{df^{o}_{c^{o}}}{f^{o}_{c^{o}}}+\frac{dP^{R}}{P^{R}}\right) \tag{21}$$

4.1.1 Tax reform in the standard Barro model

We first show that the tax reform in the standard Barro (1974) model with inelastic labor supply (fixed $\bar{T}^y$) has a negative effect on household's welfare.

Proposition 1. *At a steady-state equilibrium with no time transfer and inelastic labor supply, consider a switch from capital income tax towards inheritance tax leaving the capital-labor ratio constant. Then, first-period consumption in the market good c^y decreases, while the second-period consumption c^o increases. Moreover, steady-state life-cycle utility (20) decreases.*

Proof. Differentiating steady-state life-cycle utility $V = u\left(f^y\left(c^y, \bar{T}^y\right)\right) + v\left(f^o\left(c^o, 1\right)\right)$, and using marginal condition (14), dV has the same sign as

$$dc^y + P^R dc^o$$

Moreover, differentiating the resource constraint (18), one gets

$$c^y \frac{dc^y}{c^y} + c^o \frac{dc^o}{c^o} = 0 \tag{22}$$

Thus dV has the same sign as

$$\left(P^R - 1\right) \frac{dc^o}{c^o}$$

We now need to state the sign of dc^o. Let us define the shares of market good cost in the total cost of production of the composite good for the young $\alpha^y \equiv f^y_{c^y} c^y / f^y$ and the old $\alpha^o \equiv f^o_{c^o} c^o / f^o$. Equation (21) then rewrites as

$$\alpha^y \frac{dc^y}{c^y} - \alpha^o \frac{dc^o}{c^o} = \sigma^u \left(\frac{f^y_{c^y c^y}\left(c^y, \bar{T}^y\right) c^y}{f^y_{c^y}\left(c^y, \bar{T}^y\right)} \frac{dc^y}{c^y} - \frac{f^o_{c^o c^o}\left(c^o, 1\right) c^o}{f^o_{c^o}\left(c^o, 1\right)} \frac{dc^o}{c^o} + \frac{dP^R}{P^R} \right)$$

using the following relations:

$$\frac{df^y_{c^y}}{f^y_{c^y}} = \frac{f^y_{c^y c^y}\left(c^y, \bar{T}^y\right) c^y}{f^y_{c^y}\left(c^y, \bar{T}^y\right)} \frac{dc^y}{c^y} \text{ and } \frac{df^o_{c^o}}{f^o_{c^o}} = \frac{f^o_{c^o c^o}\left(c^o, 1\right) c^o}{f^o_{c^o}\left(c^o, 1\right)} \frac{dc^o}{c^o}$$

$$\frac{df^y}{f^y} = \alpha^y \frac{dc^y}{c^y} \text{ and } \frac{df^o}{f^o} = \alpha^o \frac{dc^o}{c^o}$$

Then, from equation (22), one easily checks that dc^o has an opposite sign to dP^R. Since the tax reform considered implies a fall in $P^R = \beta\left(1 - \tau^x\right)$, one gets $dV < 0$, which concludes the proof. □

The fall in the relative price between both intertemporal market good consumptions P^R increases the market good consumption when old c^o and pushes down the market good consumption when young c^y. Both effects are stronger when the substitutability between composite goods is important (*i.e.* high σ^u). In addition, from equation (22), the marginal rate of transformation between

c^o and c^y (MRT_{c^o/c^y}) is equal to one. As the marginal rate of substitution between c^o and c^y ($MRS_{c^o/c^y} = P^R$) is lower than the MRT_{c^o/c^y} and declines with the tax reform, household's welfare is negatively affected by the reform.

4.1.2 Tax reform with elastic labor supply

Extending the model to elastic labor supply when young modifies the effect of the tax reform, introducing labor supply effects. From equation (15), since home production functions are linear homogeneous, one deduces that the ratio c^y/T^y can be written as a function of P^y: $c^y/T^y = \phi^y(P^y)$. Since the tax reform does not modify the relative price P^y, market good consumption c^y varies in the same proportion as time devoted to home production T^y. Then any reallocation of resources from c^y to c^o is associated with a reduction in T^y by the same percentage as the reduction in c^y. One gets the following result.

Proposition 2. *At a steady-state equilibrium with no time transfer, let us consider a switch from capital income tax towards inheritance tax leaving the capital-labor ratio constant. Then, first period consumption in the market good c^y and time spent in home production T^y decrease, while the second-period consumption c^o increases. Moreover, steady-state utility increases iff*

$$C_M - \frac{P^y}{P^R} > \phi^y(P^y)\left(\frac{1}{P^R} - 1\right). \tag{23}$$

Proof. Since the home production function when young f^y is linear homogeneous and $dP^y = 0$, we deduce from marginal condition (15) that

$$\frac{dc^y}{c^y} = \frac{dT^y}{T^y} = \frac{df^y}{f^y} \text{ and } df^y_{c^y} = 0$$

Then, equation (21) rewrites as

$$\frac{dc^y}{c^y} - \alpha^o\frac{dc^o}{c^o} = \sigma^u\left(-\frac{f^o_{c^oc^o}(c^o,1)\,c^o}{f^o_{c^o}(c^o,1)}\frac{dc^o}{c^o} + \frac{dP^R}{P^R}\right)$$

Differentiating the resource constraint (18), one gets

$$(c^y + C_M T^y)\frac{dc^y}{c^y} = -c^o\frac{dc^o}{c^o} \tag{24}$$

Thus, straightforward computations lead to

$$\left[\frac{-f^o_{c^oc^o}(c^o,1)\,c^o}{f^o_{c^o}(c^o,1)}\sigma^u + \frac{c^o}{c^y + C_M T^y} + \alpha^o\right]\frac{dc^o}{c^o} = -\sigma^u\frac{dP^R}{P^R}$$

which shows that the sign of dc^o is opposite to dP^R, while dc^y and dT^y have the same sign as dP^R.

Moreover, the sign of dV is the same as

$$dc^y + P^y dT^y + P^R dc^o = (c^y + P^y T^y) \frac{dc^y}{c^y} + P^R d\frac{dc^o}{c^o}$$

Using equation (24), $dV > 0$ is equivalent to condition (23), since the tax reform considered implies a fall in $P^R = \beta (1 - \tau^x)$. $\square$

To interpret results in Proposition 2, recall that the tax reform consists in a fall in second-period consumption price P^R that increases c^o and reduces c^y and T^y. The fall in T^y improves total resources for market good consumption $C_M (1 - T^y)$ through the increase in labor supply. The positive effect of the tax reform on labor supply attenuates or dominates the Barro-model effect on utility highlighted in Proposition 1. Notice that the increase in labor supply should be stronger when the substitutability between both periods is important (*i.e.* high σ^u).

Since the capital-labor ratio is kept constant, the increase in labor supply is associated with an increase in the capital stock, and thus in savings. The young work more, consume less and then save more for their second period of life.

The consumption per additional labor unit C_M corresponds to the marginal rate of transformation between T^y and c^o $\left(MRT_{T^y/c^o}\right)$, while P^y/P^R corresponds to the marginal rate of substitution between both variables $\left(MRS_{T^y/c^o}\right)$. Then, $C_M > P^y/P^R$ means that the fall in T^y allows to produce more market goods for second-period consumption than the amount required to preserve the same welfare.

The condition $C_M > P^y/P^R$ is sufficient to guarantee welfare improvement if $P^R > 1$. But, with the initial values of the instruments that we consider ($\bar{\tau}^w \geq 0$, $\bar{\tau}^x = 0$ and $\bar{\tau}^R > 0$), the relative price P^R is equal to β, and is lower than 1. In this case, the condition $C_M > P^y/P^R$ is no longer sufficient: welfare increases if the ratio ϕ^y is small enough. Indeed, a low ϕ^y corresponds to a situation where the first-period market good consumption c^y is relatively small to T^y. Thus, the proportional reduction of c^y and T^y leads to a small reduction in c^y (small negative effect on welfare) and a sharp increase in labor supply.

In a country where people consume a large (resp. small) amount of market goods, the ratio ϕ^y would be high (resp. low) and then the tax reform would be detrimental for welfare (resp. welfare enhancing). The situation where consumption relies essentially on market goods can be associated with a developed country. By contrast, in a developing country, time devoted to home production becomes more important and consumption in market goods lower, leading to a small ratio ϕ^y. Following this interpretation, under the condition $C_M > P^y/P^R$, the tax reform is likely to be welfare enhancing in developing rather than developed countries.

4.2 Tax reform when both transfers are positive

Let us now introduce time transfers by considering the tax reform at steady state where both private transfers are positive: $x > 0$ and $T^o < 1$. Compared with the preceding section without time transfers, the marginal shift from capital income tax towards inheritance tax also modifies the parent's trade-off between bequests and time transfers. As we shall see, this adds new positive or negative effects on the young's labor supply.

The steady state is characterized by marginal conditions (14)-(16) and the resource constraint (18). In these equations, the tax reform does not only decrease the relative price P^R between both market goods consumptions, but also increases P^o, the relative price between market good and time used in home production when old. In the following, consequences of the fall in P^R will be named *interperiod* effects, while those resulting from higher P^o will be named *intraperiod* effects.

We first detail the interperiod effects. The fall in P^R has similar consequences on labor supply than those stressed in the preceding Subsection 4.1.2, but also introduces an additional effect through changes in the time transfer. Indeed, lower P^R involves a negative effect on c^y and T^y and a positive effect on c^o and T^o. The intertemporal elasticity of substitution σ^u between both composite goods may amplify these effects. The resulting impact on the young's labor supply is ambiguous: the negative effect on T^y affects positively the labor supply whereas the positive effect on T^o leads to a negative impact on time transfers, hence on the young's labor supply.

We now turn to the intraperiod effects, that come from the increase in P^o. The equality between marginal rate of substitution and relative price, $MRS_{T^o/c^o} = P^o$, implies that the marginal rate of substitution between c^o and T^o increases with the tax reform. This has a positive impact on c^o and a negative effect on T^o. The negative effect on T^o affects positively the labor supply. The magnitude of the intraperiod effect on T^o depends crucially on the elasticity of substitution between T^o and c^o. Let us denote by σ^o, the absolute value of this elasticity of substitution associated with home production technology f^o. By definition:

$$\frac{dc^o}{c^o} - \frac{dT^o}{T^o} = \sigma^o \frac{dP^o}{P^o} = -\sigma^o \frac{dP^R}{P^R} \tag{25}$$

The following Lemma signs the marginal effect on the second-period consumption c^o.

Lemma 1. *At a steady-state equilibrium with positive bequests and positive time transfers, consider a marginal switch from capital income tax towards inheritance tax leaving the capital-labor ratio constant. Then, marginal effect on second-period consumption c^o is positive and such that*

$$\frac{dc^o}{c^o} = -\left[\left(1 - \frac{c^o}{(1+\mu)\, C_M}\right)\sigma^o + \frac{c^y + C_M T^y}{(1+\mu)\, C_M}\alpha^o\left(\sigma^u - \sigma^o\right)\right]\frac{dP^R}{P^R} > 0 \tag{26}$$

where $\alpha^o \equiv f^o_{c^o} c^o / f^o$.

Proof. As the home production function when old f^o is linear homogeneous,

$$\frac{df^o}{f^o} = \alpha^o \frac{dc^o}{c^o} + (1-\alpha^o)\frac{dT^o}{T^o} = \frac{dc^o}{c^o} + (1-\alpha^o)\sigma^o \frac{dP^R}{P^R}$$

where the second equality is obtained with equation (25). Since $dP^y = 0$ and the home production function when young f^y is linear homogeneous, one deduces

$$\frac{dc^y}{c^y} = \frac{dT^y}{T^y} = \frac{df^y}{f^y} \text{ and } df^y_{c^y} = 0$$

Then, equation (21) rewrites as

$$\frac{dc^y}{c^y} - \frac{dc^o}{c^o} - (1-\alpha^o)\sigma^o \frac{dP^R}{P^R} = \sigma^u \left(-\frac{df^o_{c^o}}{f^o_{c^o}} + \frac{dP^R}{P^R} \right)$$

Linear homogeneity of f^o implies $T^o f^o_{c^o T^o}(c^o, T^o) = -c^o f^o_{c^o c^o}(c^o, T^o)$ and $\frac{-c^o f^o_{c^o c^o}}{f^o_{c^o}}\sigma^o = 1-\alpha^o$. Then, one gets

$$\frac{df^o_{c^o}}{f^o_{c^o}} = \frac{f^o_{c^o c^o} dc^o + f^o_{c^o T^o} dT^o}{f^o_{c^o}} = \frac{-c^o f^o_{c^o c^o}}{f^o_{c^o}} \sigma^o \frac{dP^R}{P^R} = (1-\alpha^o)\frac{dP^R}{P^R}$$

Consequently, the preceding relation between $\frac{dc^y}{c^y}$ and $\frac{dc^o}{c^o}$ becomes

$$\frac{dc^y}{c^y} - \frac{dc^o}{c^o} = \left[\sigma^u \alpha^o + (1-\alpha^o)\sigma^o\right] \frac{dP^R}{P^R} \tag{27}$$

Differentiation of the resource constraint (18) yields

$$c^y \frac{dc^y}{c^y} + c^o \frac{dc^o}{c^o} + C_M \left(T^y \frac{dT^y}{T^y} + \mu T^o \frac{dT^o}{T^o} \right) = 0$$

and, combining with equation (27), allows to compute dc^o/c^o:

$$\frac{dc^o}{c^o} = -\frac{(c^y + C_M T^y)\left[\sigma^u \alpha^o + (1-\alpha^o)\sigma^o\right] + \mu C_M T^o \sigma^o}{c^y + c^o + C_M (T^y + \mu T^o)} \frac{dP^R}{P^R} > 0$$

which is equivalent to equation (26). □

Lemma 1 shows that tax reform results in an increase in c^o whatever the initial values of the instruments, as soon as they allow for positive bequests and positive time transfers. We now turn to the variations of c^y, T^y and T^o that depend crucially on both elasticities of substitution σ^u and σ^o, that respectively drive up the size of the interperiod and intraperiod effects.

Lemma 2. *At a steady-state equilibrium with positive bequests and positive time transfers, consider a marginal switch from capital income tax towards inheritance tax leaving the capital-labor ratio constant. Let us assume that the initial steady state satisfies $\mu C_M > P^o$. Then, one gets the* *following sufficient conditions:*

(i) If $\sigma^o \geq \sigma^u$, the marginal effect on time devoted to home production when old T^o is negative.

(ii) If $\sigma^u \geq \sigma^o$, the marginal effects on first-period consumption in market good c^y and time devoted to home production T^y are negative.

(iii) If σ^o/σ^u is close to zero, c^y and T^y decrease, while T^o increases.

(iv) If σ^o/σ^u is close to unity, then c^y, T^y and T^o decrease.

(v) If σ^o/σ^u tends to infinity, c^y and T^y increase, while T^o decreases.

Proof. Marginal effects on c^y, T^y and T^o can be computed from expressions (25), (26) and (27):

$$\frac{dc^y}{c^y} = \frac{dT^y}{T^y} = \sigma^o \frac{c^o + \mu C_M T^o}{(1+\mu)\, C_M} \left[\alpha^o \left(\frac{\sigma^u}{\sigma^o} - 1\right) + \frac{c^o}{c^o + \mu C_M T^o}\right] \frac{dP^R}{P^R}$$

$$\frac{dT^o}{T^o} = -\, \sigma^o \frac{c^y + C_M T^y}{(1+\mu)\, C_M} \left[\alpha^o \left(\frac{\sigma^u}{\sigma^o} - 1\right) - \frac{c^o}{c^y + C_M T^y}\right] \frac{dP^R}{P^R}$$

This proves results (i)-(iv). Let us show result (v). Assuming that σ^o/σ^u tends to infinity, one gets that dc^y and dT^y are positive iff

$$\alpha^o > \frac{c^o}{c^o + \mu C_M T^o}$$

which is equivalent to $\mu C_M > P^o$, since $\alpha^o = c^o/\left(c^o + P^o T^o\right)$. The proof is complete. □

Notice that the assumption $\mu C_M > P^o$ is satisfied at the initial steady state, that is, with $\bar{\tau}^x = 0$, $\bar{\tau}^w \geq 0$ and $\bar{\tau}^R > 0$. Indeed, since $P^o = \beta\mu P^y/P^R$, straightforward calculations using linear homogeneity of the technology F show that the inequality $\mu C_M > P^o$ is always true.[9] At equilibrium, the relative price P^o is equal to the marginal rate of substitution between T^o and c^o $\left(MRS_{T^o/c^o}\right)$. Moreover, from the resource constraint, the marginal rate of transformation between T^o and c^o is: $MRT_{T^o/c^o} = \mu C_M$. Thus, the assumption $\mu C_M > P^o$ means that the MRT between T^o and c^o is higher than the MRS, that is, for given (c^y, T^y), any fall in T^o increases labor supply, and then leaves enough additional resources for second-period consumption c^o, to increase utility.

From the proof of the preceding Lemma, one may notice that increases in all variables c^y, c^o, T^y and T^o cannot arise simultaneously, since $dc^y > 0$ requires $\sigma^u < \sigma^o$, which implies $dT^o < 0$. Therefore, only three cases can arise:

[9] With $\tau^x = 0$, inequality $\mu C_M > P^o$ is equivalent to $C_M > P^y$. Using the linear homogeneity of F, one gets

$$C_M = F_L + \left[F_K - 1\right] z_M > P^y$$

where the last inequality is obtained using $\tau^w \geq 0$ and $F_K > \left(1 - \bar{\tau}^R\right) F_K = \frac{1}{\beta} > 1$.

- Case 1: $dc^y < 0$, $dT^y < 0$, $dc^o > 0$ and $dT^o > 0$. This case arises when σ^o/σ^u is close to zero. Intergenerational time transfers have been reduced by the increase in the inheritance tax.
- Case 2: $dc^y < 0$, $dT^y < 0$, $dc^o > 0$ and $dT^o < 0$. This case arises when σ^o/σ^u is close to one, as with logarithmic utility and Cobb-Douglas home production functions.[10] It induces a rise in intergenerational time transfers.
- Case 3: $dc^y > 0$, $dT^y > 0$, $dc^o > 0$ and $dT^o < 0$. This case arises when σ^o/σ^u tends to infinity. Intergenerational time transfers increase with the inheritance tax.

We now analyze the marginal effect of the tax reform on the household life-cycle utility in each of these three cases. In the following proposition, we establish the condition for the tax reform to be welfare improving.

Proposition 3. *At a steady-state equilibrium with positive bequests and positive time transfers, consider a marginal switch from capital income tax towards inheritance tax leaving the capital-labor ratio constant, the marginal effect on utility dV has the same sign as*

$$\left[P^R - \Theta\right] c^o - \alpha^o \left(\frac{\sigma^u}{\sigma^o} - 1\right) \left[(c^y + P^y T^y) - \Theta\left(c^y + C_M T^y\right)\right] \tag{28}$$

where

$$\Theta \equiv \frac{c^y + P^y T^y + P^R c^o + \beta\mu P^y T^o}{(1+\mu)\, C_M} \tag{29}$$

Proof. Using the marginal conditions of the household problem (14)-(16), dV has the same sign as

$$dc^y + P^y dT^y + P^R dc^o + \beta\mu P^y dT^o$$

Since $dP^y = 0$, relative changes dc^y/c^y and dT^y/T^y are equal. Consequently, replacing equations (25) and (27) in the latter equation and using expression (26) in Lemma 1, one obtains that dV has the same sign as

$$-\sigma^o \left[P^R - \Theta\right] c^o \frac{dP^R}{P^R} + \alpha^o \left(\sigma^u - \sigma^o\right) \left[(c^y + P^y T^y) - \Theta\left(c^y + C_M T^y\right)\right] \frac{dP^R}{P^R}$$

which concludes the proof. □

To interpret condition (28), we distinguish the above three cases according to the value of the elasticity ratio σ^o/σ^u.

[10] This is the case, for instance, if the life-cycle utility function is:

$$\alpha^y \ln c^y + (1-\alpha^y) \ln(1-T^y) + \gamma\left[\alpha^o \ln c^o + (1-\alpha^o)\ln(1-T^o)\right]$$

where α^y, α^o and γ are positive parameters, $\alpha^y < 1$ and $\alpha^o < 1$.

4.2.1 Tax reform with $\sigma^u = \sigma^o$

In this situation, that encompasses the case of a logarithmic utility function and Cobb-Douglas home production functions (as described in footnote 10), the second-period consumption c^o increases thanks to lower c^y, T^y and T^o. From expression (28), welfare increases if and only if $P^R > \Theta$, which can be rewritten as:

$$dV > 0 \Leftrightarrow C_M - \frac{P^y}{P^R} - \left(\frac{1}{P^R} - 1\right)\phi^y + (\mu C_M - P^o)\frac{T^o}{T^y} > 0$$

In the latter inequality, we observe the same term as in condition (23): $C_M - \frac{P^y}{P^R} - \left(\frac{1}{P^R} - 1\right)\phi^y$. The tax reform increases welfare in the model with elastic labor supply and no time transfer iff this term is positive. This leads to the same kind of interpretation: the fall in the second-period consumption price P^R reduces c^y and T^y and increases c^o. Then, the reduction in T^y increases the young's labor supply involving a positive effect on resources in market good.

Moreover, the positive effect on labor supply is reinforced by the increase in time transfers since T^o decreases with the reform. This positive effect on welfare appears in the second-term of the latter inequality. As stated before, the substitution from T^o to c^o is welfare enhancing since the initial equilibrium satisfies $\mu C_M > P^o$, that is, $MRT_{T^o/c^o} > MRS_{T^o/c^o}$.

Therefore, taking the Barro model with elastic labor supply as a benchmark, the introduction of intergenerational time transfers creates an additional positive effect on steady-state welfare. As soon as condition (23) is satisfied, the tax reform improves steady-state welfare. The falls in T^y and T^o involve a rise in labor supply. Simultaneously, reducing c^y and increasing c^o imply higher savings, and lead to higher capital stock. All these additional inputs allow to produce more market goods, that will be consumed in second-period of life.

4.2.2 Tax reform with $\sigma^u >> \sigma^o$

In this case, interperiod effects (from the decrease in P^R) dominate intraperiod effects (from the increase in P^o). This arises with a high elasticity of substitution between both composite goods σ^u, or with a low elasticity of substitution σ^o.

- A high σ^u involves a significant shift in resources from the first to the second period of life. Thus, the market good consumption c^o and the time devoted to home production when old T^o strongly increase thanks to lower c^y and T^y.
- For a low elasticity of substitution σ^o, the tax reform has a negative effect on time transfers. Indeed, the increase in c^o associated with strong complementarity between c^o and T^o results in an increase in T^o.

In both cases, the effect on labor supply is ambiguous as the labor supply is positively affected by the reduction in T^y and negatively by the increase in T^o. Then, the marginal effect on household life-cycle utility is less likely to be positive than in the case $\sigma^u = \sigma^o$ since the effect on labor supply is attenuated or reversed. From expression (28), the welfare is improved iff:

$$-\left[(c^y + P^y T^y) - \Theta \left(c^y + C_M T^y \right) \right] > 0$$

Using expression (29), one gets

$$C_M - \frac{P^y}{P^R} > \left(\frac{\phi^o + \mu C_M}{\phi^o + P^o} \frac{1}{P^R} - 1 \right) \phi^y \tag{30}$$

With the initial values of the instruments, we have: $\mu C_M > P^o$ and $P^R < 1$. Therefore, the difference $C_M - \frac{P^y}{P^R}$ has to be positive for the tax reform to improve welfare. Comparing inequalities (23) and (30), the right-hand side in inequality (30) is higher. Consequently, situations where the tax reform has a positive effect on welfare are less likely to happen with operative time transfers than in the Barro model with elastic labor supply. Increase in T^o reduces time transfer to the young and then affects negatively their labor supply.

4.2.3 Tax reform with $\sigma^u << \sigma^o$

Here, intraperiod effects (through higher P^o) dominate interperiod effects (through lower P^R). This case arises if σ^o is high, or if σ^u is small.

- For a high elasticity of substitution σ^o, increasing relative price P^o involves higher second-period consumption of market good c^o, lower time devoted to home production T^o, and so, higher time transfer to the young.
- A low elasticity of substitution between both periods σ^u means that both composite goods are complements. This involves a small effect of P^R and a small shift of resources from the first to the second period. This positive interperiod effect on T^o is dominated by the negative intraperiod effect created by the increase in P^o.

Since intraperiod effects dominate, the negative effect of the tax reform on time devoted to home production by the old, T^o, is strengthened. This increases first-period resources, and allows a rise in time devoted to home production by the young T^y. From the resource constraint (18), one can conclude that the tax reform results in higher labor supply since both market-good consumptions c^y and c^o increase (as stated in Lemmas 1 and 2).

The following corollary states that, despite the fall in T^o, long-run welfare always increases with the tax reform.

Corollary 1. *At a steady-state equilibrium with positive bequests and positive time transfers, consider a marginal switch from capital income tax towards inheritance tax leaving the capital-labor ratio constant. Let us assume that the initial steady state satisfies $\mu C_M > P^o$. If the ratio σ^o/σ^u tends to infinity, the marginal effect of the tax reform on utility is positive.*

Proof. Putting σ^u/σ^o at zero in condition (28), one gets that dV is positive iff

$$\left[P^R - \Theta\right] c^o + \alpha^o \left[(c^y + P^y T^y) - \Theta\left(c^y + C_M T^y\right)\right] > 0$$

Then, plugging Θ, from expression (29) into the preceding inequality yields

$$(1+\mu)\, C_M > c^y + C_M T^y + c^o + P^o T^o$$

which is true if $\mu C_M > P^o$. □

5 Transitional dynamics

In the previous section, we have analyzed the impact of the tax reform at steady state. We now rely on a numerical example in order to address the issue of intergenerational reallocation created by the tax reform by analyzing its impact on welfare along the transitional dynamics. Our aim is to exhibit a situation where a tax shift from capital income tax towards inheritance tax would be Pareto-improving. Welfare of any generation t corresponds to the infinite sum

$$W_t = \sum_{i=t}^{+\infty} \beta^{i-t} V_i$$

where V_i is life-cycle utility of generation $i \geq t$. Then a Pareto-improvement is achieved if the tax reform does not reduce W_t, for any generation $t \geq -1$, and increases W_t for at least one generation.

Before the reform, the economy is at a steady state where the capital income tax rate and the inheritance tax rate are the same as those considered in the steady-state analysis, for any $t \geq 0$: $\tau_t^R = \bar{\tau}^R > 0$ and $\bar{\tau}_t^x = 0$. Public debt is zero and public spendings $(b+g)$ are financed with a uniform tax on labor income and capital income at rate

$$\tau_t^R = \tau_t^w = \bar{\tau}^R \equiv \frac{b+g}{w\ell + Rk} > 0.$$

We also assume that, at this initial steady state, bequest and time transfers are positive.

The tax reform consists in an increase in the inheritance tax rate from zero and a fall in the capital income tax rate in order to keep the capital-labor ratio unchanged in the long run. We consider the following values of the tax instruments after the reform:

- $\tau_t^x = \tau^x > 0$ for any $t \geq 0$
- $\tau_t^R = \tau^R$ for any $t \geq 1$, where $\tau^R = 1 - \frac{1-\bar{\tau}^R}{1-\tau^x}$
- τ^w is unchanged (equal to $\bar{\tau}^R$) for any $t \geq 0$.

Under these assumptions (at this stage, we don't need to specify the respective changes in the public debt Δ_t and the second-period transfer b_t), we are able to compute the path of consumptions (c_t^y and c_t^o), time devoted to home production (T_t^y and T_t^o), and capital stock (k_t) from period $t \geq 0$.

Indeed, assuming that all constraints are satisfied, the marginal conditions of the household problem (7), (8), (9) and (10) rewrite[11]

$$\begin{cases} MRS_{c^y/c^o}(c_t^y, T_t^y, c_{t+1}^o, T_{t+1}^o) = (1-\tau^R)R_{t+1} \\ MRS_{T^y/c^y}(c_t^y, T_t^y) = (1-\tau^w)w_t \\ MRS_{c^o/c^y}(c_t^y, T_t^y, c_t^o, T_t^o) = \beta(1-\tau^x) \\ MRS_{T^o/c^o}(c_t^o, T_t^o) = \frac{\mu(1-\tau^w)w_t}{(1-\tau^x)} \end{cases} \tag{31}$$

where $w_t = F_L(k_t, \ell_t)$, $R_t = F_K(k_t, \ell_t)$ and $\ell_t = 1 - T_t^y + \mu\,(1 - T_t^o)$. Combining this set of marginal conditions with the resource constraint (12) gives the dynamical system of the equilibrium path $(c_t^y, T_t^y, c_t^o, T_t^o, k_t)_{t\geq 0}$ for a given k_0.

We consider values of some macroeconomic variables at the initial steady state that match the characteristics of the French economy. They are presented in Table 1. The resulting interest rate and wage rate are: $R = 1.7$ and $w \approx 1.47$. Assuming zero public debt, zero inheritance tax and equality between labor income tax and capital income tax leads to $\tau^w = \tau^R = 0.33$. To satisfy the modified Golden rule, the degree of altruism must be $\beta \approx 0.88$.

We use standard value for the elasticity of substitution between capital and labor ($\sigma^F = 0.5$). Normalizing production to unity, one can compute the other technological parameters: $a \approx 0.19$ and $A \approx 2.74$.

Following empirical findings cited before, we assume that time transfers have the same monetary value as cash transfers, that is, time received by a young adult from his parent multiplied by the wage rate is roughly equal to the bequest received: $\mu(1-T^o)w = x$. We keep the same value for the efficiency parameter μ as Cardia and Ng (2003): $\mu = 0.4$. Finally, as mentioned in the introduction, market good and consumption are considered as substitutes but with an elasticity of substitution

[11] We use the following definitions of the marginal rates of substitution:

$$MRS_{c^y/c^o}(c^y, T^y, c^o, T^o) \equiv \frac{u'\left(f^y(c^y, T^y)\right) f^y_{c^y}(c^y, T^y)}{v'\left(f^o(c^o, T^o)\right) f^o_{c^o}(c^o, T^o)} = \frac{1}{MRS_{c^o/c^y}(c^y, T^y, c^o, T^o)}$$

$$MRS_{T^j/c^j}(c^j, T^j) \equiv \frac{f^j_{T^j}(c^j, T^j)}{f^j_{c^j}(c^j, T^j)}, \text{ for } j = y, o.$$

Table 1: Initial steady-state values

Variable[1]	Value
$(c^y + c^o)/Y$	0.6
g/Y	0.2
k/Y	0.2
b/Y	0.13
x/Y	0.15
Rk/Y	0.34
$w\ell/Y$	0.66
ℓ	0.45

[1] $Y \equiv F(k, \ell)$.

that is not far from unity (see footnote 3): $\sigma^y = \sigma^o = 1.4$. With these elements, one can compute the share parameters $a^y \approx 0.41$ and $a^o \approx 0.51$. Finally, following again Cardia and Ng (2003), the value of the intertemporal elasticity of substitution σ^u is set to 0.25, which leads to $\gamma \approx 0.2$. Table 2 presents values of all parameters.

One can then notice that we focus here on a situation with $\sigma^o > \sigma^u$, which is quite favourable for the tax reform to be welfare enhancing in steady state. In fact, the set of parameters we consider leads to a steady state where the tax reform improves life-cycle utility if $\sigma^o \geq \sigma^u$, *i.e.* the initial steady state satisfies inequality (28) in Proposition 3.

Figure 1 gives the dynamics of consumptions, domestic times, capital stock, labor supply as well as the capital-labor ratio. The fiscal reform is Pareto-improving: it increases life-cycle utility of all generations, leading to an increase in their welfare W_t for any generation $t \geq -1$. One can notice that both consumptions rise in the long-run, as well as adult domestic time. Conversely, old-age domestic time is reduced, allowing for a rise in time transfers. This shows that the set of parameters we consider leads to Case 3 in the typology of steady-state effects we have defined in Section 4.2. Moreover, labor supply increases in all periods: the rise in time transfer is higher than the rise in adult domestic time. Moreover, capital stock also increases in all periods, allowing for higher production of market goods.

Nevertheless, the capital-labor ratio falls in the first periods. This contributes to the rise in lifetime utility of the first old (generation -1) since they benefit from a higher interest rate. Subsequent generations also experienced a fall in the wage rate. This explains the small decrease of lifetime utility between generations -1 and 0.

The effect of the tax reform on bequests and savings depends on what dynamics of the public debt the government chooses. To give an example, let us assume that the government wants to stabilize the public debt to a new steady state value from period 0 and that the second-period transfer b_t adjusts to balance government budget from period to period. Savings and bequests then converge toward finite values.[12] Figure 2 gives dynamics of public debt, savings, bequest and pension. It

[12]Indeed, savings in period $t \geq 0$ is $s_t = k_{t+1} + \Delta$, where Δ is the (constant) new public debt. Bequest x_t and

Table 2: Parameter values

Parameter		Value
Production function[a]		
Elasticity of substitution between production factors	σ^F	0.5
Scale parameter	A	2.74
Share parameter of physical capital	a	0.19
Representative household		
Home production function when young[b]		
Elasticity of substitution between c^y and T^y	σ^y	1.4
Share parameter of market good c^y	a^y	0.41
Home production function when old[c]		
Elasticity of substitution between c^o and T^o	σ^o	1.4
Share parameter of market good c^o	a^o	0.51
Preferences		
Degree of altruism	β	0.88
Efficiency of time transfer	μ	0.4
Elasticity of substitution[d] between f^y and f^o	σ^u	0.25
Time preference	γ	0.2

Note: We consider CES production and utility functions:

[a] $F(K,L) = A\left(aK^{\rho^F} + (1-a)L^{\rho^F}\right)^{\frac{1}{\rho^F}}$, with $\rho^F = 1 - \frac{1}{\sigma^F}$.

[b] $f^y\left(c^y, T^y\right) = \left(a^y\left(c^y\right)^{\rho^y} + (1-a^y)\left(T^y\right)^{\rho^y}\right)^{\frac{1}{\rho^y}}$, with $\rho^y = 1 - \frac{1}{\sigma^y}$.

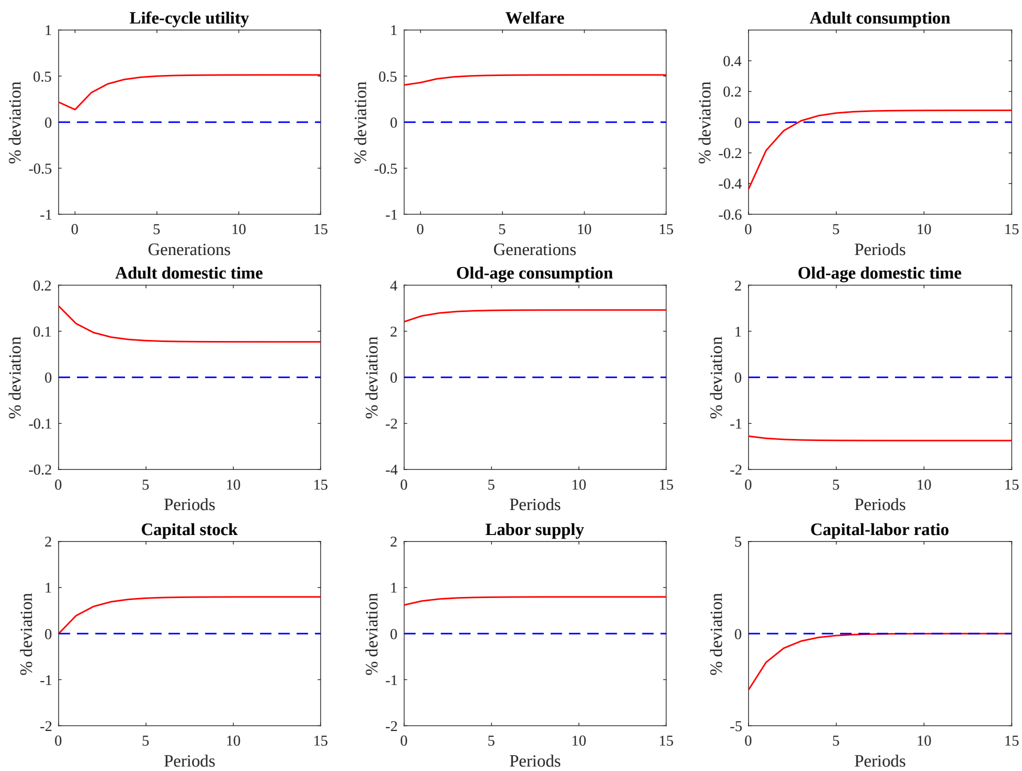
[c] $f^o\left(c^o, T^o\right) = \left(a^o\left(c^o\right)^{\rho^o} + (1-a^o)\left(T^o\right)^{\rho^o}\right)^{\frac{1}{\rho^o}}$, with $\rho^o = 1 - \frac{1}{\sigma^o}$.

[d] $u(x) = \left(1 - \frac{1}{\sigma^u}\right)^{-1} x^{1-\frac{1}{\sigma^u}}$ and $v(x) = \gamma u(x)$.

illustrates the fact that bequest can be higher after an increase in the inheritance tax. Indeed, in our example, public debt increases, which implies that savings become higher than the capital stock in each period, and the resulting bequest is higher.

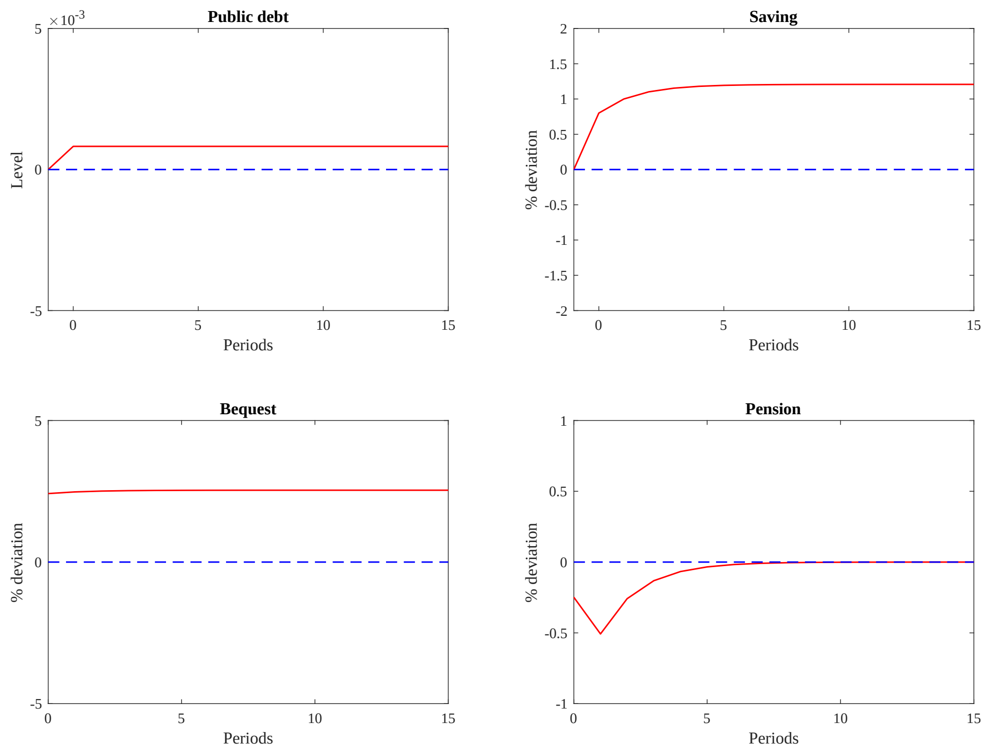
pension b_t can then be deduced from the household's budget constraint (3) and (4). From the Walras law, the sequence of pensions $(b_t)_{t\geq 0}$ obtained also balances the government budget constraint (11).

Figure 1: Consequences of the tax reform



Note: The graphics describe per-cent deviations from the initial steady-state. The inheritance tax is raised from zero to 0.03 while the capital income tax is decreased from 0.33 to approximately 0.31 in order to leave the long-run capital labor ratio unchanged.

Figure 2: Dynamics of savings and bequests



Note: The graphics describe per-cent deviations from the initial steady-state. The inheritance tax is raised from zero to 0.03 while the capital income tax is decreased from 0.33 to approximately 0.31 in order to leave the long-run capital labor ratio unchanged. The public debt is put at its new steady-state value (approximately 8.10^{-4}) from period 0.

6 Conclusion

To summarize our results, we consider a tax reform starting from an intertemporal equilibrium where the capital income tax is above its efficient level in order to finance the burden of an initial public debt. We have then addressed the following issue: should the government increase inheritance tax in order to reduce the capital income tax?

In the Barro model, the tax reform reduces steady-state welfare. The driving force is the change in the marginal rate of substitution between young and old consumptions, leading to a fall in the first-period consumption and a rise in the second-period one.

With elastic labor supply, the tax reform may be Pareto-improving. The most favorable cases are those where the fall in first-period consumption is associated with a fall in time devoted to domestic production (*i.e.* leisure in the usual terminology), allowing for an increase in the young labor supply.

With time transfers, inheritance tax also modifies the trade-off between both kinds of transfers. Grandparents are incited to transfer more time and less money to the next generation, that will benefited from higher time resources and will be able to work more. With familial time transfers, we have shown that a shift from capital income tax towards inheritance tax can be Pareto-improving. The Pareto improvement strongly depends on the strength of the positive effect of time transfers on the young's labor supply and on the strength of the effect of higher labor supply on the production of market goods.

For further research, a closer look to the intragenerational heterogeneity would allow to address redistribution issues. Heterogeneity could be introduced at least in the two following dimensions. First, empirical studies show differences in the distributions of time transfers and distributions of bequests. They suggest that bequests are more concentrated than time transfers. Secondly, capital income tax may affect a larger part of the population than inheritance tax.

7 Appendix: non-negativity constraints on bequest and time transfer

Consider

$$u\left(f^y\right) = \ln f^y \text{ and } v\left(f^o\right) = \gamma \ln f^o$$

Using linear homogeneity of f^y and f^o, marginal conditions at steady state (14)-(16) then rewrite as

$$\frac{f^y_{T^y}}{f^y_{c^y}} = P^y \Leftrightarrow \frac{c^y}{T^y} = \phi^y\left(P^y\right)$$

$$\frac{f^o_{T^o}}{f^o_{c^o}} = P^o \Leftrightarrow \frac{c^o}{T^o} = \phi^o\left(P^o\right)$$

$$\frac{v' f^o_{c^o}}{u' f^y_{c^y}} = P^R \Leftrightarrow \gamma \frac{T^y}{T^o} \frac{f^o_{c^o}\left(\phi^o, 1\right)/f^o\left(\phi^o, 1\right)}{f^y_{c^y}\left(\phi^y, 1\right)/f^y\left(\phi^y, 1\right)} = P^R \Leftrightarrow \frac{T^y}{T^o} = \psi\left(\frac{P^R}{\gamma}, P^y, P^o\right)$$

We express conditions for positive transfers with respect to budget shares

$$\alpha^y = \frac{c^y}{c^y + P^y T^y} = \frac{\phi^y}{\phi^y + P^y} \qquad \text{and} \qquad \alpha^o = \frac{c^o}{c^o + P^o T^o} = \frac{\phi^o}{\phi^o + P^o}$$

Since $P^R P^o = \beta\mu P^y$, we get

$$\phi^y = \frac{\alpha^y P^y}{1-\alpha^y}, \qquad \phi^o = \frac{\alpha^o \beta\mu P^y}{\left(1-\alpha^o\right) P^R} \qquad \text{and} \qquad \psi = \frac{\beta\mu}{\gamma}\frac{1-\alpha^y}{1-\alpha^o} \tag{32}$$

Lemma A.1. *With logarithmic form for u and v, one gets the following equivalence*

$$\begin{aligned}
x > 0 \quad &\Leftrightarrow \quad \left(\alpha^y + \frac{\gamma\alpha^o}{P^R}\right)\frac{P^y}{C_M} \\
&\quad > \frac{1+\mu-\frac{g}{C_M}}{1+\mu+\frac{P^R b}{C_M}}\left(\gamma\alpha^o + 1 + \frac{\gamma\left(1-\alpha^o\right)}{P^R}\right) + \alpha^y - \left(1 + \frac{\gamma\left(1-\alpha^o\right)}{P^R}\right) &(33)\\
T^o < 1 \quad &\Leftrightarrow \quad \beta\mu > \frac{\gamma\left(1-\alpha^o\right)}{\frac{P^y}{C_M}\left(\alpha^y + \frac{\gamma\alpha^o}{P^R}\right) + \left(1-\alpha^y\right)}\left(1 - \frac{g}{C_M}\right) &(34)\\
T^y > \mu\left(1-T^o\right) \quad &\Leftrightarrow \quad \beta\mu < \frac{\beta\left(1-\alpha^y\right) + \gamma\left(1-\alpha^o\right)}{\frac{P^y}{C_M}\left(\alpha^y + \frac{\gamma\alpha^o}{P^R}\right)}\left(1 - \frac{g}{C_M}\right) &(35)
\end{aligned}$$

where $P^R = \beta\left(1-\tau^x\right)$ and

$$\frac{C_M}{P^y} = \frac{1}{1-\tau^w}\left(1 + \left[\frac{1}{\beta\left(1-\tau^x\right)\left(1-\tau^R\right)} - 1\right]\frac{z_M}{F_L\left(z_M, 1\right)}\right)$$

Proof. The resource constraint (18) rewrites

$$T^o\left[\left(\phi^y + C_M\right)\psi + \phi^o + \mu C_M\right] = C_M\left(1+\mu\right) - g$$

and allows to compute T^o

$$T^o = \frac{1+\mu}{\frac{\beta\mu}{\gamma(1-\alpha^o)}\left[\alpha^y \frac{P^y}{C_M} + 1 - \alpha^y + \gamma\alpha^o \frac{P^y}{P^R C_M}\right] + \mu} \tag{36}$$

Then, condition $T^o < 1$ is equivalent to (34).

The intertemporal budget constraint of a consumer (19) allows to compute the steady-state bequest x

$$\left[\left(\phi^y + P^y\right)\psi + P^R\left(\phi^o + \beta^{-1}P^o\right)\right] T^o = P^y\left(1+\mu\right) + P^R b + \left(1-\tau^x\right)\left(1-\beta\right)x$$

Recalling that $P^R P^o = \beta\mu P^y$ and using (32), we get that bequest x is positive iff

$$\left[\frac{\beta}{\gamma\left(1-\alpha^o\right)} + \frac{\alpha^o\beta}{1-\alpha^o} + 1\right]\mu T^o > 1 + \mu + \frac{P^R b}{P^y}$$

Replacing T^o by its expression in (36) leads to condition (33). Finally, using (32), condition $T^y > \mu\left(1-T^o\right)$ rewrites

$$\left(\frac{\beta\left(1-\alpha^y\right)}{\gamma\left(1-\alpha^o\right)} + 1\right) T^o > 1$$

Replacing T^o with its expression in (36) leads to (35). This concludes the proof. □

Consider a case with $\alpha^y = \alpha^o$ and $g = b = 0$. The condition for nonnegative bequest (33) is exactly the same as the one we get in the standard Barro model with inelastic labor supply and no time transfer. Indeed, let us assume $\tau^w = \tau^R = \tau^x = 0$, we get

$$\frac{C_M}{P^y} = 1 + \left(\frac{1}{\beta} - 1\right)\frac{z_M}{F_L\left(z_M, 1\right)}$$

and the condition for positive bequest becomes

$$z_M > \frac{\gamma}{1+\gamma} F_L\left(z_M, 1\right)$$

The latter is the same as the condition derived by Thibault (2000) in the Barro model. It states that positive bequest are obtained at steady state if, at the modified Golden-rule, savings in the corresponding Diamond economy is lower than the capital stock.

Condition for non negative time transfer (34) is likely to be satisfied for high values of μ (high efficiency of the time transfer from the old in home production of the young) or if α^o is closed to one (home production when old relies heavily on market good consumption). Nevertheless, μ cannot be too high in order to avoid situations where labor supply of the young is larger than one.

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