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"Gains from Intra-Firm Trade and Multinational Production"

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Abstract

This paper quantifies the welfare gains from intra-firm trade. We propose a model where firms have access to competing market strategies: export and multinational production. Due to technological appropriability issues, foreign affiliates import an intermediate input from the home headquarters. The presence of export and multinational production alters the standard results obtained for welfare in heterogeneous firm models, through a double truncation of the productivity distribution. The model is then calibrated to analyze counterfactual scenarios. We find that welfare gains from intra-firm trade range from 0.3 to 7 percent depending on country characteristics.

JEL classification: F12, F23

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1 Introduction

Engaging in international trade is an exceedingly rare activity: In 2000, only 4 percent of all U.S. firms were exporting (Bernard et al. (2007), Eaton et al. (2011) among others). In this context, multinational firms play a key role: 90 percent of U.S. exports and imports occur through them with 50 percent of U.S. imports occurring within the same firm rather than through arms length (Bernard et al. (2009)). Moreover, several empirical studies conveys the idea that intra-firm trade is mainly related to the transfers of capabilities within the corporation. For example, Ramondo et al. (2013) find that most U.S. foreign affiliates are not created for multistage production chains, but as outputs to produce and then supply in the local market. Similarly, Atalay et al. (2014) study domestic operations of U.S. multinational firms, and provide evidence of intra-firm transfers of intangible inputs.

We embed knowledge transfer in Helpman et al. (2004) to build a model of export and horizontal multinational production with intra-firm trade. In our general equilibrium framework with N asymmetric countries, each foreign affiliate imports an intermediate input from the home country due to technological appropriability issues. Therefore, an important activity of our multinational firm is to transfer capabilities or knowledge-intensive inputs from the home headquarter to the foreign affiliate. This mechanism renders the knowledge-intensive input used in multinational production mobile across regions. Moreover, this implies that geographical costs apply to both exports and multinational production because they involve transportation of a finished good and of an intermediate good, respectively.

An increase in trade barriers affects the multinational production strategy in two different ways. First, sales of the existing foreign affiliates decrease, which generates a new margin of adjustment for multinational firms.

This occurs because of the complementarity between export and multinational activities. In models where export and multinational activities are substitutes this margin disappears, since the sales of existing affiliate firms are not directly affected by a change in trade policy. Second, trade barriers increase the threshold productivity cutoff for multinationals: the need to import intermediate goods from the headquarter makes it more difficult to enter as a foreign affiliate when trade costs increase. Firms face an altered proximity-concentration trade-off because their choice to maintain capacity in other markets crucially depends on trade costs and not only on the forgone economies of scale. By contrast, in Helpman et al. (2004), an increase in trade costs unambiguously makes the multinational production strategy more attractive. In their framework, the proximity-concentration trade-off arises from the fact that only exported goods are subjected to iceberg transport costs, while multinational activity is free of trade cost.

Our contribution is threefold. Firstly, the presence of export and multinational production alters the standard results obtained for welfare in heterogeneous firm models, through a double truncation of the productivity distribution. Upon drawing its own efficiency parameter, each firm decides whether to exit or to produce. In the latter case, the firm must face additional fixed costs linked to the supply strategy chosen. When the firm decides to serve the foreign market, it chooses whether to export domestically produced goods or to produce abroad via affiliate production. The presence of two alternative ways of reaching the foreign location introduces a double truncation in the productivity distribution of exporters. This affects average export sales which are now dependent on firm's productivity level. Furthermore, it implies that domestic trade share and trade elasticity are no longer sufficient statistics to evaluate welfare gains.[1]

[1]Hence, the results in Arkolakis et al. (2008) and Arkolakis et al. (2012) do not apply when multinational production is added to the model.

Secondly, we exploit the absence of free entry to retrieve gravity equations and compare margins' sensitivity for exports and affiliate sales with respect to alternative models such as Chaney (2008) and Helpman et al. (2004). As regards the margin of exports, we find that, similarly to Chaney (2008), the intensive margin only depends on the elasticity of substitution. Differently from Chaney (2008), the extensive margin is a function of both export and affiliate sales. With respect to the margins of affiliate sales, we show that the intensive margin is unambiguously related to the elasticity of substitution and the share of the imported intermediate good; whereas, the sensitivity of the extensive margin depends on the the endogenous wage rate, making it harder to interpret analytically.

Thirdly, we quantify the country level gains from multinational production with intra-firm trade. We use data on bilateral trade (BACI) and intra-firm flows (BEA) for France and the U.S., to calibrate the model and analyze counterfactual scenarios. Our findings stress the role of intra-firm trade for welfare gains: they range from 0.3 to 7 percent depending on country characteristics. We also compare the total gains from a model of multinational production and intra-firm with a model of pure multinational production. The latter yields the largest rise in welfare due to trade liberalization. Finally, we compute the sensitivity of export and affiliate sales, confirming the important role of the elasticity of substitution for both modes of supply.

This paper relates to several strands of literature. As in Horstmann and Markusen (1992), Brainard (1997), and Helpman et al. (2004), we capture the interaction between export, multinational production, and intra-firm trade. Keller and Yeaple (2012) measure the spatial barriers to transferring knowledge. They find that the knowledge intensity of production affects the level of affiliate sales around the world. Irarrazabal et al. (2012) structurally estimate a model of trade and multinational production with firm heterogeneity. They reject the proximity versus concentration hypothesis which did not consider intra-firm trade. Corcos et al. (2012), using French firm-level data, investigate the main determinants of the internalization choice. Their findings highlight the role of capital, skill and productivity in explaining the choice of intra-firm trade.

This paper also contributes to the growing literature that theoretically analyzes the welfare gains from openness. Arkolakis et al. (2012) show that there exists a group of models in which a country's domestic trade share and the elasticity of trade are sufficient statistics to measure aggregate welfare gains from trade. This result relies on the assumption of an unbounded productivity distribution. Feenstra (2013) uses a bounded Pareto distribution and non CES preferences to restore the role for product variety and pro-competitive gains from trade in heterogeneous firm models. Melitz and Redding (2013) show that the additional adjustment margin in heterogeneous firm models plays an important role for welfare gains. Differently from Arkolakis et al. (2012) and similarly to Feenstra (2013) and Melitz and Redding (2013), our welfare measure is altered by the double truncation in the productivity distribution of exporters. This makes our welfare measure depending on endogenous wages, and not only on domestic trade share and trade elasticity.

Another related strand of literature quantifies the gains from multinational production. Ramondo (2014) uses a multi-country general equilibrium model with a continuum of goods produced under constant return to scale at the industry level to calculate the gains that a country would experience from liberalizing access to foreign firms. Ramondo and Rodriguez-Clare (2013) consider trade and multinational production into an Eaton-Kortum framework to measure the overall gains from openness. Garetto (2013) quantifies the gains from multinational activity, using an Eaton-Kortum type model, where multinational firms engage in vertical

FDI. Irarrazabal et al. (2012) find that impeding multinational activity has a small effect on welfare. Similarly to most of these studies, we propose a mechanism through which intra-firm trade affects multinational production, and rely on aggregate evidence to quantify its importance.

The rest of the paper is organized as follows. Section 2 provides a description of facts on U.S. multinational firms and U.S. exports. Section 3 describes the theoretical framework. Section 4 characterizes its general equilibrium and derives intensive and extensive margins. In Section 5, we investigate the theoretical implications of the model on welfare. Section 6 contains the calibration. Finally, Section 7 concludes.

2 Trade, Multinational Production and Distance: a First Look

We bring together data on exports and horizontal multinational production (henceforth MP) to account for regularities such as market entry, volume of affiliate and export sales, and geographical distance.[2] All figures are relegated to the Appendix, and variables are expressed in logarithms.

Market Entry

Figure 2a plots the number of U.S. affiliates selling to a market against the market size for 55 destination countries.[3] The number of firms selling to a market increases with the size of the market. The relationship is neater in Figure 2b where the number of U.S. affiliates is normalized to the share of U.S. affiliate sales in a market. The X-axis of Figure 2b reports market size across all destinations, and the Y-axis shows the ratio between the number of U.S. affiliates in a given market and the U.S. market share. The U.S. market share is defined as total U.S. affiliate sales to that market $X^M_{us,j}$

[2] Appendix A describes the data.

[3] The list of countries is in Table 6.

divided by market j's GDP X_j

$$\pi_{us,j} = \frac{X^M_{us,j}}{X_j}. \tag{1}$$

This relationship is tight. As in Eaton et al. (2011), when we correct for market share, Canada becomes a negative outlier. The regression line has a positive slope of 0.73.

Figures 2a and 2b show that the number of sellers in a market varies with market size, which supports the use of a model of firm heterogeneity, where the number of firms depends on country size.

Affiliate and Export Sales

Figures 3 and 4 report U.S. affiliate and export sales for our sample of 55 destination countries. As shown in Figure 3, total affiliate and export sales are increasing in destination country size: regression lines have slopes of 0,77 and 0.57, respectively. Figure 4 shows the negative relationship between affiliate and export sales with respect to distance when controlling for size. A regression between affiliate sales (exports) and distance to destination countries suggests a negative relationship with a slope coefficient of -0,42 (-0,67).

This evidence confirms the existence of geographical costs affecting export and MP strategies.

Intra-Firm trade

Figure 5a plots the relationship between U.S. exports of goods shipped to affiliates by U.S. parents, by country of affiliate, against total affiliate sales in a given market. Figure 5b shows the increase in the value of goods sold by the U.S. affiliate in country j: all points lie below the 45 degree line.

This supports the idea of an imported intermediate channel.

3 Theoretical Framework

In this section we propose a model of export, MP, and intra-firm trade without free entry which accounts for the regularities described in Section 2. This framework allows us to study the supply mode choice between MP and export in a multi-country framework.[4]

3.1 Preferences

The preferences of a representative consumer are given by a CES utility function over a continuum of final goods indexed by v

$$U = \left[\int_{v \in V} c(v)^{(\sigma-1)/\sigma} dv \right]^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where $\sigma > 1$ represents the elasticity of substitution between any two goods within the group, and V is the set of available varieties.

3.2 Supply

There are N potentially asymmetric countries. Country n has a population L_n whose labor supply is inelastic.

There is one differentiated sector which produces a continuum of horizontally differentiated varieties, $q(v)$, from two intermediate goods, y_1 and y_2. Both y_1 and y_2 are produced with one unit of labor, but y_1 can only be made at home, due to technological appropriability issues. This assumption is crucial for MP strategy: y_1 can be considered as transfer of capabilities between the headquarter and the foreign affiliate. Each variety is then supplied by a Dixit-Stiglitz monopolistically competitive firm which produces under increasing returns to scale originating from a fixed cost. We assume

[4] A model with only intra-firm trade is sufficient for the purpose of our study. Notice that this model will be isomorphic to a model accounting for both intra-firm trade and MP fixed cost depending on distance.

that the fixed cost is paid in units of labor of the country where the good is produced.

We consider three modes of supply in the differentiated sector: (i) firms which sell only domestically; (ii) firms which export; and, (iii) firms which supply the foreign market via MP. Hence, when a firm decides to serve a foreign market, it chooses whether to export domestically produced goods or to locate production via an affiliate in the foreign country. In making these decisions, firms compare the net profits from exports and MP.

In our model, the classical scale versus proximity trade-off is altered by the introduction of intra-firm trade, which makes the MP strategy sensitive to geographical frictions between countries. The fact that y_1 can only be produced at home plays an important role. If a firm chooses to supply the foreign market via local sales of its affiliates, the affiliate must import the intermediate good y_1 from the home country. This implies that the MP strategy does not entirely avoid geographical related costs. The trade link between the home parent and the affiliate captures the complementary relationship between export and MP.

Upon drawing its own parameter a from a cumulative density function $G(a)$ that is common to every country, each firm decides whether to exit (if it has a low productivity draw), or to produce. In the latter case, the firm faces additional fixed costs linked to the mode of supply chosen: (i) if it chooses to produce only for the domestic market i, it pays the fixed market entry cost, f_{ii}; (ii) if it chooses to export, it bears the additional costs f_{ij}^X of meeting different market specific standards (e.g., the cost of creating a distribution network in a new country j); finally, (iii) if the firm chooses to serve foreign markets through MP, it will bear a fixed costs f_{ij}^M, which is a combination of the fixed cost of creating a distribution network, and the fixed cost of building up new capacities in the foreign country.[5] We allow

[5]In our model, if a firm chooses to serve foreign markets via MP, the local foreign

for the fixed costs to differ across countries.

3.3 Intermediate Results

3.3.1 Demand

The CES utility function implies that the demand of a representative consumer from country i for a good of type a is given by

$$c_i(a) = A_i p_i(a)^{-\sigma} \quad \text{with } A_i \equiv \frac{Y_i}{P_i^{1-\sigma}}, \tag{3}$$

where a denotes the unit labor coefficient, A_i is the demand shifter, $p_i(a)$ is the final price of a variety produced by a firm with marginal cost a, and P_i is the CES price index of the final good. A_i is exogenous from the perspective of the firm: it is given by the ratio of the aggregate level of spending on the differentiated good Y_i, and the CES price index $P_i^{1-\sigma}$.

3.3.2 Organization and Product Variety

We assume that the production of the final good combines two intermediate goods, y_1 and y_2, in the following Cobb-Douglas function

$$q_i(a) = \frac{1}{a}\left(\frac{y_1}{\eta}\right)^{\eta}\left(\frac{y_2}{1-\eta}\right)^{1-\eta} \quad 0 < \eta < 1 \tag{4}$$

where $1/a$ represents the firm specific productivity parameter, and η is the Cobb-Douglas cost share of y_1, common across all countries. When trade is possible, firms decide whether to sell to a particular market. The supply mode (export or MP) will depend on their own productivity, the trade costs between the origin and the destination country, and the fixed costs.

The marginal costs in the exporting sector will be higher than in the

affiliate will produce the intermediate good y_2 only. Then y_2 will be combined with the intermediate good imported from the headquarter, y_1.

FDI sector. Since y_1 and y_2 are produced with labor, L, the marginal cost for domestic as well as export production is linear in τ

$$mc_{ij} = aw_i\tau_{ij}, \tag{5}$$

where $\tau_{ij} = 1$ when $i = j$. The marginal cost for supplying the foreign market j via local sales of foreign affiliates is concave in τ

$$mc_{ij}^M = aw_j^{1-\eta}\left(w_i\tau_{ij}\right)^{\eta}. \tag{6}$$

mc_{ij}^M combines inputs (i.e., labor) from home and host country. More precisely, $w_j^{1-\eta}$ is the labor cost of an input produced in country j, while w_i^{η} is the labor cost of an the input imported by country j from the home country i.[6] Note that in this framework trade costs matter only in relation to the share of intermediate good y_1 that is used in the production of final good η. Using the mark-up $\sigma/\left(\sigma-1\right)$, we can easily derive the price for each particular mode of supply.

3.3.3 Mode of Supply Decisions

The choice of the mode of supply is made by comparing various profit levels. We can distinguish three relevant cases:

Case (i) If a firm decides not to supply a market and exits, its operating profits are zero.

Case (ii) If a firm in country i decides to supply market j via exports, the profits from exporting to market j are decreasing in τ_{ij} in a linear fashion

$$\pi_{ij}^X = \left[p_{ij}\left(a\right) - aw_i\tau_{ij}\right]q\left(a\right)_{ij} - w_jf_{ij}^X, \tag{7}$$

[6]Further details for cases where $\eta = 1$ (as in Chaney (2008)) and $\eta = 0$ (as in Helpman et al. (2004)) are provided in Appendix E.

where $q(a)_{ij}$ denotes the quantity exported. Substituting the equilibrium price and quantity we have

$$\pi_{ij}^{X} = \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{(1-\sigma)} Y_j \left(w_i a \tau_{ij}\right)^{1-\sigma} / P_j^{1-\sigma} - w_j f_{ij}^{X}, \tag{8}$$

where the fixed cost of exporting f_{ij}^{X} is evaluated at the foreign wage rate w_j.[7]

Case (iii) If a firm in country i decides to supply market j via affiliate sales, the profits realized by a subsidiary located in the j country depend on τ_{ij}

$$\pi_{ij}^{M} = \left[p^{M}(a) - a w_j^{1-\eta}\left(w_i \tau_{ij}\right)^{\eta}\right] q(a)_{ij}^{M} - w_j f_{ij}^{M}, \tag{9}$$

where $q(a)_{ij}^{M}$ represents the quantity supplied by the foreign affiliate. Substituting the equilibrium price and quantity, we obtain

$$\pi_{ij}^{M} = \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{(1-\sigma)} Y_j \left(a w_j^{1-\eta}\left(w_i \tau_{ij}\right)^{\eta}\right)^{1-\sigma} / P_j^{1-\sigma} - w_j f_{ij}^{M}, \tag{10}$$

where τ_{ij}^{η} is the trade costs associated with the intermediate good y_1, that is imported from the home country. The foreign affiliate has to face both the fixed cost f_{ij}^{M}, evaluated at the foreign wage rate, and the trade costs (that hit the imported intermediate).

We set parameters to get the same ranking as in Helpman et al. (2004). Namely, only firms with sufficiently high productivity will supply the foreign market, with the most productive firms supplying it via MP rather than exports. Hence, the regularity condition is

$$f_{ij}^{X} < f_{ij}^{M} w_j^{(1-\eta)(\sigma-1)} \left(w_i \tau_{ij}\right)^{(\eta-1)(\sigma-1)}. \tag{11}$$

Since the price index depends on the probability distribution, we have to

[7]Note that this mode of supply collapses to domestic production when $i = j$, since $\tau_{ii} = 1$.

assume a particular functional form for $G(a)$ in order to obtain closed-form solutions. Following the empirical literature on firm size distribution, we assume that the unit labor requirements are drawn from a Pareto distribution. The cumulative distribution function of a Pareto random variable a is

$$G(a) = \left(\frac{a}{a_0}\right)^k, \tag{12}$$

where k and a_0 are the shape and scale parameters, respectively. The shape parameter k represents the dispersion of cost draws. An increase in k implies a reduction in the dispersion of firm productivity-draws. Hence, the higher is k, the smaller is the amount of heterogeneity.

The support of the distribution $[0, ..., a_0]$ is identical for every country, and a_0 represents the upper bound of the distribution. The productivity distribution of surviving firms will also be Pareto with shape k. More precisely, since a firm will start producing only if it has at least a productivity of $1/a_{ij}$, the probability distribution of supplying as an exporter, or as a foreign affiliate, is conditional on the probability of successful entry in each market

$$G(a/a_{ii}) = \left(\frac{a}{a_{ii}}\right)^k. \tag{13}$$

The above truncated cost distribution exploits the fractal nature of the Pareto's, with support $[0, ..., a_{ii}]$. Given the assumed parameterization, we can explicitly solve for the price index.

Following Chaney (2008), we assume that the total mass of potential entrants in country i is proportional to its labour income, L_i. Hence, larger and wealthier countries have more entrants. The absence of free entry implies that firms generate net profits which are redistributed to workers (or shareholders), proportionally to each own share w_i of the global fund.

3.3.4 Demand for Differentiated Goods

Total income in country j, Y_j, is computed as the sum of workers' labor income in country j, $w_j L_j$, and the dividends from their portfolio, $\pi w_j L_j$, where π is the dividend per share. Given the optimal pricing of firms and the consumers' demand, the export value from country i to country j for a firm with unit labor requirement a is equal to

$$x_{ij}^X = p_{ij}^X q_{ij}^X = Y_j \left(p_{ij}^X\right)^{1-\sigma} / P_j^{1-\sigma}, \tag{14}$$

where $p_{ij}^X = [\sigma / (\sigma - 1)] a w_i \tau_{ij}$ and $q_{ij}^X = \left(p_{ij}^X\right)^{-\sigma} \beta Y_i / P_j^{1-\sigma}$. Affiliate sales by a firm located in j are

$$x_{ij}^M = p_{ij}^M q_{ij}^M = Y_j \left(p_{ij}^M\right)^{1-\sigma} / P_j^{1-\sigma}, \tag{15}$$

where P_j represents the price index of good q in country j. We can observe that the values of export and total production in j's foreign affiliates are similar to those derived in a setting with homogeneous firms. We now have the basis for gravity equations of export and affiliate sales.

Since only firms with $a \leq a_{kj}$ will produce, the price index in country j is[8]

$$P_j^{1-\sigma} = \sum_{k=1}^{N} w_k L_k \left[\int_0^{a_{kj}^M} \left(w_j^{1-\eta} \left(w_k \tau_{kj}\right)^{\eta}\right)^{1-\sigma} a^{1-\sigma} dG(a) + \int_{a_{kj}^M}^{a_{kj}} \left(w_k \tau_{kj}\right)^{1-\sigma} a^{1-\sigma} dG(a) \right]. \tag{16}$$

[8]Since we are not conditioning by $G(a/a_{ij})$, the number of firms will be the number of entrants and not the number of active firms. Moreover, we consider a_{ij} to be the unit labor requirement for exporting. Note that when $i = j$, $\tau_{ii} = 1$. Therefore $a_{ij} = a_{ii}$ corresponds to the cutoff of domestic firms.

The dividend per share, π, is defined as

$$\pi = \frac{\sum\limits_{k,l=1}^{N} w_k L_k \left[\int\limits_{0}^{a_{kl}^{M}} \pi_{kl}^{M} dG(a) + \int\limits_{a_{kl}^{M}}^{a_{kl}} \pi_{kl} dG(a) \right]}{\sum\limits_{n=1}^{N} w_n L_n}, \tag{17}$$

where in the square bracket we have the profits that a firm with a specific threshold level in country k earns from a specific mode of supply in country l.[9] A similar analysis can be extended to H sectors. In Appendix C we derive solutions for the profits.

4 Equilibrium with Heterogeneous Firms

To compute the equilibrium of the overall economy, we solve for the selection of firms into different modes of supply. We generate predictions for aggregate bilateral trade and affiliate sales.

4.1 Productivity Threshold

The productivity threshold of the least productive firm in country i that exports to country j is

$$a_{ij}^{1-\sigma} = \lambda_1 \frac{w_j f_{ij}^{X}}{Y_j} \frac{P_j^{1-\sigma}}{(w_i \tau_{ij})^{1-\sigma}}, \tag{18}$$

where $\lambda_1 = \sigma^{\sigma} (\sigma - 1)^{(1-\sigma)}$.[10]

The productivity threshold of the least productive firm in country i which opens a foreign affiliate in country j, is obtained by equating the operating profits from doing MP in equation (9), to the operating profit

[9] Note that when $i = j$, we are considering domestic firms.

[10] We interpret $a^{1-\sigma}$ as a measure of productivity.

from exporting in equation (8):

$$(a_{ij}^M)^{1-\sigma} = \lambda_1 \frac{w_j \left(f_{ij}^M - f_{ij}^X\right)}{Y_j} \frac{P_j^{1-\sigma}}{\left(w_j^{1-\eta}\left(w_i\tau_{ij}\right)^\eta\right)^{1-\sigma} - \left(w_i\tau_{ij}\right)^{1-\sigma}}. \tag{19}$$

4.2 Equilibrium Price Indices

Since the price index adjusts depending on country characteristics, and the number of potential entrants n_E is exogenously given, we obtain

$$\begin{aligned} P_j^{1-\sigma} &= \left(\frac{k}{k-\sigma+1}\right)\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\sum_{k=1}^{N} w_k L_k \left[(a_{kj}^M)^{k-\sigma+1}\left(w_j^{1-\eta}\left(w_k\tau_{kj}\right)^\eta\right)^{1-\sigma}\right. \\ &\quad \left. + \left(a_{kj}^{k-\sigma+1} - (a_{kj}^M)^{k-\sigma+1}\right)\left(w_k\tau_{kj}\right)^{1-\sigma}\right]. \end{aligned}$$

Plugging the productivity thresholds from (18) and (19) into the above expression, we can solve for the price index in the destination country j as follows

$$P_j = \lambda_2 Y_j^{\frac{b-1}{b(1-\sigma)}} \theta_j \left(\frac{Y}{1+\pi}\right)^{\frac{1}{b(1-\sigma)}}, \tag{20}$$

where $b = k/(\sigma-1)$, $\lambda_2^{b(\sigma-1)} = (\sigma/(\sigma-1))^{\sigma-1}[(k-\sigma+1)/k]\lambda_1^{b-1}$, w_k is the wage paid to workers in country k by exporting firms, and w_j is the wage paid to the workers who are either producing the j-domestic varieties or the intermediate good y_2 used by the foreign affiliate in country j.[11] θ_j in equation (20) collects the following terms

$$\begin{aligned} \theta_j^{b(1-\sigma)} &= \sum_{k=1}^{N}\frac{Y_K}{Y}\left[\left(w_j\left(f_{kj}^M - f_{kj}^X\right)\right)^{1-b}\left[\left(w_j^{1-\eta}\left(w_k\tau_{kj}\right)^\eta\right)^{1-\sigma} - \left(w_k\tau_{kj}\right)^{1-\sigma}\right]^b\right. \\ &\quad \left. + \left[w_j f_{kj}^X\right]^{1-b}\left(\left(w_k\tau_{kj}\right)^{1-\sigma}\right)^b\right], \end{aligned} \tag{21}$$

where Y is the world output. θ_j is an aggregate index of j's remoteness from the rest of the world, and it can be thought as the "multilateral trade

[11] Appendix D provides detailed derivations of the price index.

resistance" introduced by Anderson and van Wincoop (2003). It takes into consideration the role of fixed and trade costs as well as the intermediate input traded.

Since total income Y will depend on the dividends received from the global fund, in equilibrium the amount of dividends per share is a constant.

4.3 Equilibrium Variables

The mode of supply choice depends on each firm's productivity, the trade costs, the aggregate demand, the amount of intermediates, and the set of competitors. Plugging the general equilibrium price index (20) into the productivity thresholds (18) and (19), we can solve for the equilibrium productivity thresholds:

$$\bar{a}_{ij}^{1-\sigma} = \lambda_4 \frac{w_j f_{ij}^X}{(w_i \tau_{ij})^{1-\sigma}} \theta_j^{1-\sigma} \left(\frac{Y}{Y_j}\right)^{\frac{1}{b}} (1+\pi)^{-\frac{1}{b}}, \tag{22}$$

$$(\bar{a}_{ij}^M)^{1-\sigma} = \lambda_4 \frac{w_j \left(f_{kj}^M - f_{ij}^X\right)}{\left(w_j^{1-\eta} (w_i \tau_{ij})^{\eta}\right)^{1-\sigma} - (w_i \tau_{ij})^{1-\sigma}} \theta_j^{1-\sigma} \left(\frac{Y}{Y_j}\right)^{\frac{1}{b}} (1+\pi)^{-\frac{1}{b}}, \tag{23}$$

where $\lambda_4 = \lambda_1/\lambda_2^{\sigma-1}$ is constant. The productivity threshold in (22) is unambiguously positively affected by the wage rate in the origin country, and distance trade costs. The productivity threshold in (23) is ambiguously affected by the wage rate in i, the intensity in headquarter services η, and trade costs.

The share of imported intermediates plays an important role in determining the substitutability or the complementarity between export and MP strategies. For low intensity in imported intermediate (low η), the MP strategy becomes more attractive when trade costs increase, making MP and exports substitutes. On the contrary, higher level of η makes MP and export complements, so that both activities require a higher productivity when trade barriers increase.

Using the demand function, the equilibrium price, and the price index (20), we can find firm level exports, firm level affiliate sales, aggregate output, and dividends per share π:

$$x_{ij}^X = p_{ij}^X q_{ij}^X = \lambda_3 \theta_j^{\sigma-1} \left(\frac{Y_j}{Y}\right)^{\frac{1}{b}} (1+\pi)^{\frac{1}{b}} \left(w_i \tau_{ij}\right)^{1-\sigma} a^{1-\sigma}, \tag{24}$$

$$x_{ij}^M = p_{ij}^M q_{ij}^M = \lambda_3 \theta_j^{\sigma-1} \left(\frac{Y_j}{Y}\right)^{\frac{1}{b}} (1+\pi)^{\frac{1}{b}} \left(w_j^{1-\eta} \left(w_i \tau_{ij}\right)^{\eta}\right)^{(1-\sigma)} a^{1-\sigma}, \tag{25}$$

$$\pi = \lambda_5, \tag{26}$$

$$Y_j = (1+\pi) w_j L_j = (1+\lambda_5) w_j L_j, \tag{27}$$

where $\lambda_3 = \lambda_2^{\sigma-1}(\sigma/(\sigma-1))^{1-\sigma}$ and $\lambda_5 = \left((1-\lambda_4^{-b}\sigma)/\sigma\right) / \left(1-(1-\lambda_4^{-b}\sigma)/\sigma\right)$ are constants. Equations (24)-(27) are functions of: country size L_j, wages, trade barriers τ_{ij}, fixed costs f_{ij}^M and f_{ij}^X, proportions of imported intermediate, η, and measures of the j's location with respect to the rest of the world θ_j.

Similar to Chaney (2008), exports by individual firms depend on the trade cost τ_{ij} with an elasticity of $(1-\sigma)$. Additionally, we characterize sales by a foreign affiliate: they depend on the share of intermediate y_2 produced in the foreign location, and on the intermediate y_1 imported from the home country. Intra-firm trade implies that firm level affiliate sales in (25) is unambiguously affected by trade costs: an increase in trade costs reduces firm level affiliate sales. The behaviour of a single firm is similar to what a traditional model of trade and MP with representative firms would predict for aggregate bilateral trade flows and affiliate sales.

Using (24) and (25), we can derive gravity equations. In this model aggregate bilateral trade and overseas affiliate sales will behave differently from traditional models.

Proposition 1 *(Aggregate Exports Sales) Total export (f.o.b.)* X_{ij}^X *from country* i *to country* j *are*

$$X_{ij}^X = \frac{Y_i Y_j}{Y}\theta_j^{b(\sigma-1)}\left(w_i\tau_{ij}\right)^{1-\sigma}\left[\left(\frac{w_j f_{ij}^X}{(w_i\tau_{ij})^{1-\sigma}}\right)^{1-b} - \left(\frac{w_j\left(f_{ij}^M - f_{ij}^X\right)}{\left(w_j^{1-\eta}(w_i\tau_{ij})^{\eta}\right)^{1-\sigma} - (w_i\tau_{ij})^{1-\sigma}}\right)^{1-b}\right]. \quad (28)$$

Proof. See Appendix E.1. ■

The gravity equation (28) suggests that exports are a function of country sizes Y_i and Y_j, wages, bilateral trade and fixed costs, and the measure of j's remoteness from the rest of the world.

Differently from Chaney (2008), this expression for aggregate trade takes into consideration the interaction between export and MP. This interaction makes the gravity for export non linear in logarithm. We expect aggregate export sales to decrease with trade costs, and this decrease should be faster the larger is σ. This is reduced for large value of imported intermediate. However, since in (28) wages are endogenous, and thus respond to changes in parameters of the model, the overall effect is ambiguous.

Proposition 2 *(Aggregate Affiliate Sales) Total affiliate sales* X_{ij}^M *in country* j *are*

$$X_{ij}^M = \frac{Y_i Y_j}{Y}\theta_j^{b(\sigma-1)}\left(w_j^{1-\eta}\left(w_i\tau_{ij}\right)^{\eta}\right)^{1-\sigma}\left[\frac{w_j\left(f_{ij}^M - f_{ij}^X\right)}{\left(w_j^{1-\eta}(w_i\tau_{ij})^{\eta}\right)^{1-\sigma} - (w_i\tau_{ij})^{1-\sigma}}\right]^{1-b}. \quad (29)$$

Proof. See Appendix E.2. ■

The gravity equation (29) suggests that affiliate sales are a function of country sizes Y_i and Y_j, wages, bilateral trade and fixed costs, intra-firm trade intensity, and the measure of j's remoteness from the rest of the world.

Depending on the intensity of imported headquarter intermediates, η, an increase in trade barriers might create an incentive to ship production to the foreign market to avoid a part of the trade costs. This increases the demand for labor in the destination country relative to the home country.

When the difference between wages is not too big, an increase in trade barriers can lead to a raise in aggregate local sales. This effect is stronger the lower is the share of intra-firm trade. Since in general equilibrium the increase in trade costs will also affect wages, the overall effect of trade policy is ambiguous.

4.4 Intensive and Extensive Margins

In what follows we derive margins for export and affiliate sale equations.

4.4.1 Affiliate Sales

In this section we examine the intensive and extensive margins of affiliate sales. We analyze how the elasticity of substitution as well as the share of intermediate inputs affects the sensitivity of these margins. Differentiating total affiliate sales in country j, $X_{ij}^M = w_i L_i \int_0^{\overline{a}_{ij}^M} x_{ij}^M dG(a)$, with respect to τ_{ij}, we derive the intensive and extensive margins of affiliate sales

$$\frac{\partial X_{ij}^M}{\partial \tau_{ij}} = \underbrace{w_i L_i \int_0^{a_{ij}^M} \frac{\partial x_{ij}^M}{\partial \tau_{ij}} dG(a)}_{\text{Intensive Margin}} + \underbrace{w_i L_i x_{ij}^M G'\left(\overline{a}_{ij}^M\right) \frac{\partial \overline{a}_{ij}^M}{\partial \tau_{ij}}}_{\text{Extensive Margin}}, \tag{30}$$

where we applied the Leibniz rule to separate the margins.

Proposition 3 *Defining $\psi \equiv -\partial \ln X_{ij}^M / \partial \ln \tau_{ij}$, a change in variable costs τ_{ij} makes the margins of affiliate sales react in the following way:*

$$\psi = \underbrace{\eta\left(\sigma - 1\right)}_{\textit{Intensive Margin Elasticity}} \underbrace{+ (k - \sigma + 1) \frac{\left(\eta \left(w_j^{1-\eta} \left(w_i \tau_{ij}\right)^{\eta}\right)^{1-\sigma} - \left(w_i \tau_{ij}\right)^{1-\sigma}\right)}{\left(w_j^{1-\eta} \left(w_i \tau_{ij}\right)^{\eta}\right)^{1-\sigma} - \left(w_i \tau_{ij}\right)^{1-\sigma}\right)}}_{\textit{Extensive Margin Elasticity}}. \tag{31}$$

Proof. See Appendix F.1. ■

Intensive Margin. The intensive margin of affiliate sales depends on the constant elasticity of substitution and on the level of imported intermediate η. Therefore, when goods are very substitutable (high σ), the sales of each individual affiliate is very sensitive to the trade barriers. Let us now focus the role of the parameter η.

When $\eta = 1$, no firm will supply via MP. In this case, the foreign affiliate is importing both intermediate inputs from the home country. This strategy is extremely costly, since it implies full trade costs as well as higher fixed costs. Therefore, when $\eta = 1$ export is the only market access strategy.

Differently, when $\eta = 0$, the foreign affiliate is producing using only foreign inputs (similarly to Helpman et al. (2004)). When all intermediates are realized in the foreign location, the volume of sales of already existing affiliates are not affected by changes in trade costs. Therefore, in this case the intensive margin elasticity is equal to zero.

For intermediate levels of η, both the extensive and the intensive margins of affiliate sales are affected by the intensity of imported headquarter intermediates. The behaviour of the intensive margin is unambiguous: σ magnifies the sensitivity of the intensive margin. When σ is high, the change in X_{ij}^M due to a change in τ is mostly captured by the intensive margin: if τ decreases, new affiliates enter the market, but a high σ leads to a high level of competition. In this environment, having a low productivity is an even bigger disadvantage as firms can only capture a small market share, and their impact on the overall affiliate sales is small.

Extensive Margin. The sensitivity of the extensive margin of affiliate sales to changes in trade costs, is strictly related to the elasticity of substitution σ. In general, we should expect that when the substitutability across varieties is low, an increase in σ makes entrance of new affiliates more sensitive to changes in τ. On the one hand, trade liberalization makes easier to import the intermediate goods; on the other hand, the low degree of substitution keeps the level of competition down. This explains why more firms can survive as new affiliates after entry. Contrarily, a larger degree of substitutability among varieties makes entry of new affiliates less sensitive to changes in τ. In fact, when the level of competition is high, new entrants will capture only a small fraction of market share despite the reduction in trade costs.

Notice that equation (31) responds to changes in wages, which are endogenously determined. We leave to the calibration section the general equilibrium analysis of how trade policy affects the sensitivity of affiliate sales.

4.4.2 Exports Sales

In this section we examine the intensive and extensive margins of export sales. After differentiating the expression of total exports in country j, $X_{ij}^X = w_i L_i \int_{\bar{a}_{ij}^M}^{\bar{a}_{ij}} x_{ij}^X dG(a)$, with respect to trade costs, we derive the intensive and extensive margins of export sales

$$\frac{\partial X_{ij}^X}{\partial \tau_{ij}} = \underbrace{w_i L_i \int_{\bar{a}_{ij}^M}^{\bar{a}_{ij}} \frac{\partial x_{ij}^X}{\partial \tau_{ij}} dG(a)}_{\text{Intensive Margin}} + \underbrace{w_i L_i \left[x_{ij}^X G'\left(\bar{a}_{ij}\right) \frac{\partial \bar{a}_{ij}}{\partial \tau_{ij}} - x_{ij}^M G'\left(\bar{a}_{ij}^M\right) \frac{\partial \bar{a}_{ij}^M}{\partial \tau_{ij}} \right]}_{\text{Extensive Margin}}, \tag{32}$$

where we applied the Leibniz rule once again to separate the margins.

Proposition 4 *Defining $\Omega \equiv -\partial \ln X_{ij}^X / \partial \ln \tau_{ij}$, a change in the variable costs τ_{ij} makes the margins of export sales to react as follows:*

$$\Omega = \underbrace{(\sigma - 1)}_{\textit{Intensive Margin Elasticity}} + \underbrace{(k - \sigma + 1) \left[1 - \frac{X_{ij}^M}{X_{ij}^X} (\Gamma - \omega) \right]}_{\textit{Extensive Margin Elasticity}}, \tag{33}$$

where

$$\Gamma = \frac{\eta \frac{\left(w_j^{1-\eta}(w_i\tau_{ij})^{\eta}\right)^{1-\sigma}}{\tau_{ij}} - \frac{(w_i\tau_{ij})^{1-\sigma}}{\tau_{ij}}}{\left(w_j^{1-\eta}\left(w_i\tau_{ij}\right)^{\eta}\right)^{1-\sigma} - \left(w_i\tau_{ij}\right)^{1-\sigma}}, \quad (34)$$

$$\omega = \left(\frac{w_i\tau_{ij}}{w_j}\right)^{(1-\eta)(1-\sigma)}. \quad (35)$$

Proof. See Appendix F.2. ■

Intensive Margin. Similarly to Chaney (2008), the volume of export sales depends on the constant elasticity of substitution. This implies that when goods are very substitutable (high σ), the export of each exporter is very sensitive to the trade barriers.

Extensive Margin. When variable trade barriers move, the extensive margin behaves quite differently than in Chaney (2008). In our model, the sensitivity of the extensive margin of exports to trade policy depends on the interaction between aggregate affiliate and export sales.[12] This is because the change in the number of varieties supplied via exports depends on the level of profits generated by the export and MP strategies, which in turns affect overall affiliate sales.

Let us focus on the second part of (33). If $X_{ij}^M > X_{ij}^X$, a decrease in trade cost reduces the extensive margin elasticity of export. Notice that the sign of the overall elasticity depends on the size of the intensive margin, which can compensate the negative extensive margin. When $X_{ij}^M < X_{ij}^X$ the opposite is true, and a decrease in trade costs increases the extensive margin. To summarize, while the elasticity of the intensive margin is always positive (a decrease in trade costs increases the volume of trade), the behaviour of the extensive margin depends on how export and affiliate sales

[12]Note that $\Gamma > \omega$ is true for certain parameter restrictions consistent with our calibration. For further details on Γ and ω, see appendix F.

interact.

Differently from Chaney (2008), the elasticity of exports and affiliate sales with respect to variable costs depends on the elasticity of substitution between goods σ. This result, which is discussed more carefully in the next section, suggests that countries' asymmetries embedded in a multi supply framework are relevant to fully understand how variable costs affect bilateral flows. This framework reaffirms the importance of σ even allowing for firm heterogeneity. To further stress this result, in the calibration section we proposes an exercise to understand the effects of trade policy on both export and affiliate sales' margins.

5 Welfare

In this model the welfare of each representative consumer is given by $C_j = w_j/P_j$, which does not depend on the assumption of free entry. We follow the procedure suggested by Arkolakis et al. (2008) to obtain an expression for the domestic trade share λ_{jj}, and the wage w_j.

We start by deriving the average sales for export, affiliate sales, and domestic firms, which results in the following equations:

$$(\overline{p_{ij}q_{ij}})^X = \left(\frac{k\sigma}{k-\sigma+1}\right)\left(\frac{a_{ij}^{k-\sigma+1} - (a_{ij}^M)^{(k-\sigma+1)}}{a_{ij}^{1-\sigma}[a_{ij}^k - (a_{ij}^M)^k]}\right) w_j f_{ij}^X, \tag{36}$$

$$(\overline{p_{ij}q_{ij}})^M = \left(\frac{k\sigma}{k-\sigma+1}\right) w_j f_{ij}^M, \tag{37}$$

$$(\overline{p_{jj}q_{jj}})^D = \left(\frac{k\sigma}{k-\sigma+1}\right) w_j f_{jj}. \tag{38}$$

Equations (37) and (38) are standard with respect to the literature, and independent of the productivity levels. On the contrary, average sales of exporting firms in equation (36) are functions of marginal costs, a. This happens despite the assumptions of Pareto distribution and Dixit-Stiglitz preferences. Therefore, the result in (36) differs from Arkolakis et al. (2008) and Melitz and Redding (2013).

Few remarks can be made on equation (36). First, the second term, i.e., the ratio including the difference in the cutoffs, is lower than 1. This makes average export sales smaller than in models with only exporting firms. This effect is driven by the additional level of competition characterizing our set up. Second, average export sales decline with a reduction in average productivity of exporters, while it increases with a reduction in average productivity of multinationals.[13]

Equations (36), (37) and (38) are used to obtain total export and affiliate sales from country i to j as well as domestic sales in country j. Total export sales from country i to j are

$$T_{ij}^X = \underbrace{w_i L_i [a_{ij}^k - (a_{ij}^M)^k]}_{\text{no. exporting firms}} \underbrace{\left(\frac{k\sigma}{k-\sigma+1}\right)\left(\frac{a_{ij}^{k-\sigma+1} - (a_{ij}^M)^{(k-\sigma+1)}}{a_{ij}^{1-\sigma}[a_{ij}^k - (a_{ij}^M)^k]}\right) w_j f_{ij}^X}_{\text{avg. exporting sales}}. \tag{39}$$

Total affiliate sales from country i to j are

$$T_{ij}^M = \underbrace{w_i L_i (a_{ij}^M)^k}_{\text{no. MP firms}} \underbrace{\left(\frac{k\sigma}{k-\sigma+1}\right) w_j f_{ij}^M}_{\text{avg. affiliate sales}}. \tag{40}$$

From equations (39) and (40) we can obtain total sales in country j as

$$\sum_v T_{vj} = \sum_v T_{ij}^X + \sum_v T_{vj}^M, \tag{41}$$

where it is worth stressing that both T_{vj}^X and T_{vj}^M include domestic sales. We are now able to compute the domestic trade share, which is given by

$$\lambda_{jj} = \frac{T_{jj}}{\sum_v T_{vj}}. \tag{42}$$

[13]The proofs can be found in Appendix G.

From equation (42) we can obtain an expression for w_j as a function of domestic trade share λ_{jj}. This result will allow us to express welfare w_j/P_j in terms of domestic trade share, trade elasticity, wages and parameters.

Since in our set up, average export sales depends on productivity, the expression for welfare turns out to be complex and highly non linear. More specifically, endogenous variables like w_j and w_v are left inside the welfare expression. This is different from models with only exporters, where welfare would be a function of domestic trade share, trade elasticity and parameters. We conclude that in models where alternative market strategies occur simultaneously, a country's domestic trade share and trade elasticity are no longer sufficient statistics to evaluate welfare gains.[14]

To have a better understanding of what happens to welfare, we propose a calibration exercise to evaluate the effects of trade liberalization.

6 Calibration

The general equilibrium structure of the model provides the framework to welfare analysis. Moreover, the absence of free entry enables us to derive and study the sensitivity of intensive and extensive margins of exports and affiliate sales to specific parameters in the model. We start by describing the calibration exercise and then we move to analyzing gains from intra-firm trade and margins' sensitivity.

We calibrate the model to match volumes of trade and multinational activities for France relative to the U.S. With the calibrated model, we quantify the gains from intra-firm trade. Counterfactual experiments show how the gains depend on the degree of competition of the market and on the extent of barriers to multinational production.

[14]This result is obtained even without introducing sequential production as in Melitz and Redding (2013).

We start by describing the calibration of the parameters in the model. We identify the Home country with France (FR), and the Foreign country with the U.S. (US). Calibration of the bilateral model requires to assign values to the parameters of the production functions η and σ, to the shape parameter k, to trade costs τ, and to the size parameter L_{us}.[15]

Data on multinational firms are obtained from the BEA website, which provides information on inward and outward direct investment.[16] Among different types of information provided by BEA, this paper will focus on relationships occurring with Majority-owned Nonbank Foreign Affiliates. The calibration exercise involves two measures of multinational production: U.S. intra-firm exports of goods to France affiliates (intra-firm exports from U.S. parent to its French affiliate), and French intra-firm exports of goods to U.S. affiliates. Unfortunately, due to data restriction we have not access to French intra-firm data. To overcome this limitation, we use imports by U.S. parents from their French affiliates as a proxy for French MNFs' activity in the U.S. Data on export are taken from the CEPII dataset (HS6 1992 classification). To obtain a more reliable measure of arm's-length trade, we subtract intra-firm trade from export values.

Calibration requires, first, to find the solution of the model and, second, to find parameters that minimize the distance between the moments generated by the model and those computed from the data. The solution of the model is found by solving a system of six equations and six unknowns: equations (22) and (23), for each of the two countries to determine their productivity thresholds; and two labor market clearing conditions to determine the equilibrium wage of the U.S., and the equilibrium value of the France labor force. The wage in France is normalized to 1.

We choose the shape parameter of the Pareto distribution to be $k = 4$.

[15]Notice that we assume pairwise symmetric trade costs between countries.

[16]See Appendix A for further details on the data used.

The elasticity of substitution σ is a measure of product differentiation and market power, and has a large effect on the computation of the welfare gains. We take the elasticity of substitution, $\sigma = 4$, from Broda and Weinstein (2006): over the 1999-2001 period they find average and median elasticities for SITC 5-digit goods of 13.1 and 2.7, respectively (see their Table IV). The value $\sigma = 4$ implies a mark-up of 33 percent. Geographical and trade barriers are set to $\tau = 1.5$, which is in line with the estimate of 1.7 in Anderson and van Wincoop (2003). For the value of η, we follow the findings in the literature and assign a magnitude of 1/3 to intra-firm trade.

The remaining parameters are jointly calibrated to match a set of relevant moments in the data. We choose six parameters related to fixed costs, $PAR = \{f_{us}, f_{fr}, f^X_{us,fr}, f^X_{fr,us}, f^M_{us,fr}, f^M_{fr,us}\}$ to match the ratio of wages w_{fr}/w_{us}, the labor efficiency units in France relative to the U.S. L_{fr}/L_{us}, the French share of U.S. exports, the intra-firm share of exports of French multinational corporations to their U.S. affiliates relative to the U.S., the ratio of exports relative to the ratio of GDPs, the ratio of FDIs relative to the ratio of GDPs.

The vector of calibrated parameters is a vector

$$\widehat{PAR} = \arg\min_{PAR} \sum \left[mom - \widehat{mom}(PAR)\right]^2,$$

where mom is the vector of moments from the data, and $\widehat{mom}(PAR)$ is the vector of moments generated by the model as function of the vector of parameters PAR. The calibrated parameters are in Table 1. All matched data are for the year 1999 and are listed in Table 2.

The baseline calibrated model implies that the fixed costs of intra-firm exports from French MNFs' to the U.S. is less than a half lower than the fixed costs born by U.S. MNFs' firms.[17] This is necessary to match a level

[17]This implies that American firms have a higher productivity than French firms,

Table 1: Calibrated Parameters

Parameter		Value
France domestic fixed cost	f_{fr}	0.058
U.S. domestic fixed cost	f_{us}	0.020
France export fixed cost	$f^X_{fr,us}$	0.400
U.S. export fixed cost	$f^X_{us,fr}$	0.390
France intra-firm export fixed cost	$f^M_{fr,us}$	1.100
U.S. intra-firm export fixed cost	$f^M_{us,fr}$	3.500

Table 2: Moments targeted in the estimation

Calibration target	Data	Model	Data Source
Ratio of wages w_{fr}/w_{us}	0.71	0.81	OECD
Ratio of population L_{fr}/L_{us}	0.21	0.21	CEPII gravity dataset
Ratio of exports	1.10	1.18	CEPII gravity dataset
Ratio of intra-firm trade	0.50	0.50	BEA
Share of exports (Ratio of exports/ Ratio of GDPs)	6.81	6.80	CEPII gravity dataset
Share of intra-firm trade (Ratio of intra-firm trade/ Ratio of GDPs)	2.90	2.90	CEPII and BEA

of French intra-firm exports which is less than half the level of U.S. intra-firm export. The U.S. fixed cost of export $f^X_{us,fr}$ is slightly lower than the French one. Its value $f^X_{us,fr} = 0.39$ is in line with the estimate of 0.545 in Melitz and Redding (2013).

6.1 Welfare Gains

Welfare gains are computed as changes in real wages obtained from a counterfactual equilibrium where parameters have been perturbed.[18]

We start by computing the gains from intra-firm trade (*GIF*) by com-

allowing the former to bear a much higher fixed cost.

[18]Note that in this model, both wages and price index are endogenous. Therefore, they both respond to variations in parameters.

paring the calibrated economy and a counterfactual world with only export activity. Precisely, they are the ratio between the welfare in the calibrated economy and the welfare in the economy where $\eta \to 1$ in both countries.[19] The results are in Table 3.

Table 3: Welfare Gains from Intra-Firm (GIF)

	Baseline Calibration	$\sigma = 3.5$	$\sigma = 4.15$	$\eta = 0.67$	$\eta = 0.28$	$k = 3.8$
U.S. welfare gains	1.069	1.031	1.090	1.052	1.070	1.097
France welfare gains	1.003	0.993	1.007	0.945	1.015	1.004

The first column shows the welfare gains in the calibrated economy. As the fixed costs of MP are higher for the U.S. than for French firms, the gains from opening to intra-firm trade are higher for the U.S. than France. Intra-firm trade among American firms in France allows final good producers to use more productive technologies and to pay lower wages. Moreover, the possibility of integration reduces the prices charged on traded intermediates.

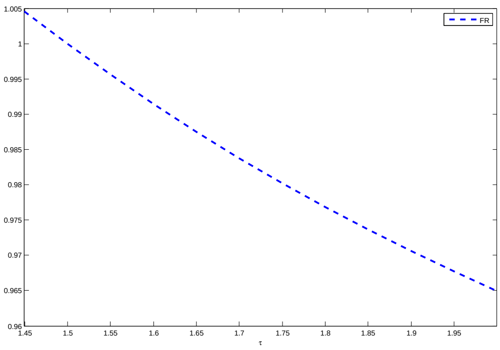
In the second and third columns, we report the same computation but changing σ. First, the elasticity of substitution is lowered to 3.5 which increases the market power to 40 percent. The welfare gains driven by intra-firm trade decrease for both countries because both price indices increase but in the U.S. the wage rate increases by a larger amount. Second, we show that a lower market power of 31 percent generates a substantial increase in welfare when the economies open to intra-firm trade. In columns four and five, we show the impact of a lower and higher share of intra-firm trade on welfare. A higher η induces a welfare loss in France because of the high fixed costs related to MP. Lastly in columns six, we analyze the effect of

[19]When $\eta \to 1$, both intermediates will be produced in the home headquarter. This makes the MP strategy very costly, so that no firm will ever choose the MP strategy. In this scenario, the set up collapses to domestic and exporting firms.

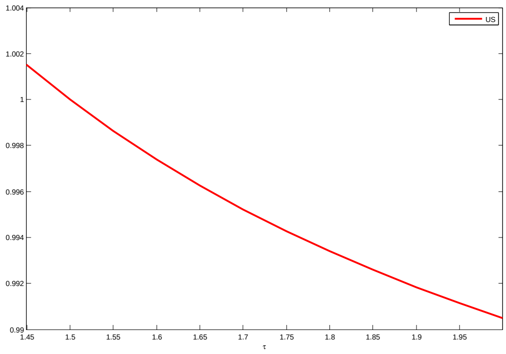
changes in productivity dispersion, k. We consider a situation in which the economies are characterized by higher productivity dispersion (lower k), which implies a larger number of high productivity firms. In this case, welfare gains driven by intra-firm trade are larger but mainly concentrated in the U.S.

In Figure 1, panels 1a and 1b, the welfare gains (or losses) are computed as the ratio between the welfare of the economy with the calibrated parameters but $\tau \in [1.45, 2]$, and the welfare of the same economy with $\tau = 1.5$. We can observe that the decrease in relative welfare is more important for France than for the U.S. The higher French productivity in export activity turns into a larger disadvantage in case of increasing trade frictions.

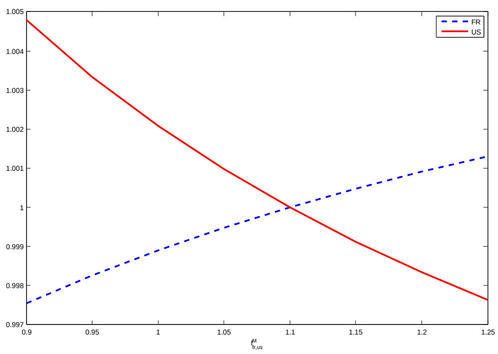
Figure 1: Welfare Gains from MP with Intra-Firm Trade



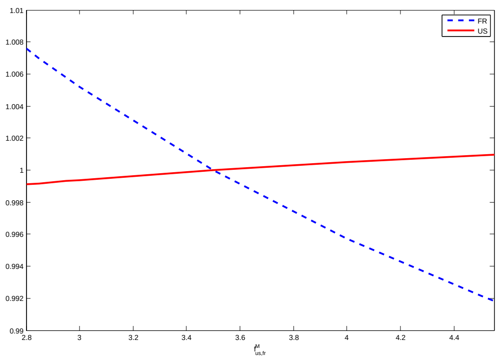
(a) France - Variable Cost



(b) U.S. - Variable Cost



(c) Fixed Cost $f^M_{fr,us}$



(d) Fixed Cost $f^M_{us,fr}$

In the bottom panels we show the welfare gains (or losses) related to changes in the fixed cost of MP, other things being equal. In this exercise, each fixed cost of MP, $f^M_{us,fr}$ or $f^M_{fr,us}$, is set to its calibrated value, i.e. 3.5 or 1.1, respectively. We can observe the asymmetric reactions of countries. Consider panel 1c in Figure 1. When $f^M_{fr,us}$ increases, everything else equal, welfare in France increases while it decreases in the U.S. A raising $f^M_{fr,us}$ implies that French MP's strategy has become more costly: less French firms will be able to reach the foreign destination via MP. On the one side, the higher cost of reaching the U.S. foreign market lowers the number of French varieties entering the U.S. market via MP. Therefore, the U.S. market experiences a reduction in welfare. On the other side, the rise in $f^M_{fr,us}$, makes some of the less productive MNFs to switch to export activity. Since French firms are more efficient in export activity, these new exporting firms will hire workers thereby increasing real wage (relatively to U.S.). In terms of magnitude, France benefits more than the U.S. from an increase in the fixed cost of MP (see the dotted line in panel 1c and the solid line in panel 1d).

In Table 4, we report the welfare gains from pure multinational production (GMP), i.e. horizontal MP with no intra-firm trade. This implies to consider a model where $\eta = 0$. The absence of intra-firm flows makes this model to capture the proximity versus concentration hypothesis. In each column, the gains are computed as the ratio between the welfare from the economy with export and pure MP and the economy where only exports are allowed. The parameters of the model are taken from the calibration in the first column and modified in the rest of the table.

Table 4: Welfare Gains from Pure Multinational Production (GMP)

	Baseline Calibration	$\sigma = 3.5$	$\sigma = 4.15$	$k = 3.8$
U.S. welfare gains	1.045	1.025	1.054	1.058
France welfare gains	1.103	1.046	1.121	1.121

Table 4 shows larger welfare gains for France than for the U.S. This might be due to the level of trade costs used in the calibration exercise. Since these are not sufficiently high to make MP activity very convenient, export seems to be the preferred activity.

According to our exercise, France will always gain from MP, but relatively more from pure MP. In fact, comparing Table 3 to Table 4, we can observe that the gains are higher for France when intra-firm trade is not included in the model, but the contrary is true for the U.S. The reason is that in our calibrated economy, France turns out to be less efficient than U.S. in multinational activity with intra-firm linkages. Conversely, the U.S. has the largest welfare gains when MP with intra-firm trade is considered.

From Tables 3 and 4 we can conclude that the overall gains from trade is larger in pure MP models. In fact, pure MP corresponds to a case in which the Pareto distribution is less constrained than in MP with intra-firm.

6.2 Intensive and Extensive Margins

Concerning the margins, we exploit this model with alternative market strategies to quantify the impact of trade liberalization on the sensitivity of intensive and extensive margins. We compute each margin considering several parameter values other than the benchmarks'. This enables us to quantify differences in margins sensitivity with respect to alternative models, such as Chaney (2008) and Helpman et al. (2004).

Since the purpose of this paper is to evaluate the importance of intra-firm trade, we only report results for affiliate sales. A similar analysis can be provided for extensive margin of exports. Nevertheless, the interaction between export and FDI in equation (33) makes the interpretation very difficult. The results are in Table 5, where the margins are computed using equation (31). The first line shows the relative wage when the parameters are modified.

Table 5: Affiliate Sales - Extensive and Intensive Margins

	Benchmark	$\sigma = 3.5$	$\sigma = 4.15$	$\eta = 0.67$	$\eta = 0.28$	$k = 3.8$
Relative Wage $1/w_{us}$	0.81	0.855	0.792	0.831	0.810	0.762
Affiliate Sales - FRANCE						
Intensive Margins	1	0.833	1.050	2	0.840	1
Extensive Margins	0.139	0.100	0.134	0.567	0.068	0.116
Affiliate Sales - U.S.						
Intensive Margins	1	0.833	1.050	2	0.840	1
Extensive Margins	0.100	0.036	0.101	0.569	0.021	0.080

As the model predicts, σ magnifies the sensitivity of the intensive margins of affiliate sales. This is line to what happens to export: when goods are very substitutable, i.e. σ is high, the export of each individual exporter is very sensitive to τ. Table 5 also shows that the larger is the share of imported intermediate, η, the larger will be the response of affiliate sales to changes in trade barriers.

The results for the extensive margins are more complex. In the U.S., the extensive margins of affiliate sales become more sensitive to trade barriers when σ is high. This result is different with respect to what happens to the extensive margin of export. This stands in the characteristic of affiliate sales which rely only partially on exports, via the parameter η. When variable trade barriers go down, some of the most productive U.S. exporting firms can switch to a MP mode of supply. In turn, these marginal top exporters will have a substantial impact on aggregate affiliate sales. On the contrary, a larger σ generates an additional disadvantage for French multinational firms: the increase in competition makes it harder for French firms to enter as multinational.

The other parameters affect the two countries in a similar way. The higher is the intensity in imported intermediates, η, the more sensitive will be this extensive margin to changes in τ. Lastly, when k is small, the extensive margin is less sensitive to changes in trade barriers. This happens because when firms' productivity is sufficiently dispersed (k is low), competition is tougher due to the presence of a larger number of high productivity firms. Thus, when trade barriers decrease there is a smaller number of potential candidates for MP.

7 Conclusion

Our goal in this paper has been to evaluate welfare and gains from intra-trade in a general equilibrium model with export and multinational production. We have assumed that each foreign affiliate imports an intermediate input from the home country due to technological appropriability issues. This set up captures the interaction between alternative market access strategies, by allowing the knowledge-intensive input used in multinational production to move over geographical space. Therefore, geographical costs apply to both exports and multinational production because they involve transportation of a finished good and of an intermediate good, respectively.

We have investigated the effects of an increase in trade barriers on multinational production. First, it increases the productivity cutoff: the need to import intermediate goods from the headquarter makes it more difficult to enter as a foreign affiliate when trade costs increase. Second, sales of the existing foreign affiliates decrease, which implies the existence of a new margin of adjustment for multinational firms.

An important theoretical result of the paper is that the presence of alternative market access strategies alters the standard results obtained for welfare in heterogeneous firm models, through a double truncated productivity distribution. Our model shows that with export and multinational production, the welfare gains from trade are also affected by the response of wages to changes in trade costs.

To quantitatively assess the country level gains from multinational production with intra-firm trade, we calibrate the model to match aggregate U.S. and French data. Our findings stress the role of intra-firm trade for additional welfare gains: they range from 0.3 to 7 percent depending on country characteristics. Moreover, we exploit the delivered gravity equations to compare margins' sensitivity for exports and affiliate sales with respect to alternative models such as Chaney (2008) and Helpman et al. (2004). Our framework reaffirms the importance of the elasticity of substitution in models with firm heterogeneity.

Extensions of the model should be devoted to the introduction of a free entry condition. However, we believe the analysis conducted here is a useful starting point to understand the mechanism governing the firms' decisions about sourcing, and of the welfare consequences of having multiple market access strategies.

Appendix - For Online Publication

Appendix A provides data information, and appendix B presents some stylized facts. Appendices C-F provide derivations for equilibrium variables and proofs of the propositions.

A Data

Data on inward and outward direct investment, including data on direct investment positions and transactions and on the financial and operating characteristics of the multinational companies are available from the U.S. Bureau of Economic Analysis.[20] Our sample includes all the majority-owned nonbank affiliates, i.e. foreign affiliates in which the combined direct and indirect ownership interest of all U.S. parents exceeds 50 percent. All information concern the manufacturing industry.

The preliminary evidence in section 2 relies on data for U.S. parents and their foreign affiliates in the benchmark year 2004. Data used include: number of U.S. foreign affiliates in different destination countries; local affiliate sales; volume of U.S. intra-firm trade, i.e. U.S. Exports of Goods Shipped to Affiliates by U.S. Parents, by Country of Affiliate. Data on export are taken from the Center for International Data at UC Davis. We focus on U.S. export flows for 2004. To make trade data comparable with BEA data, we reduce the sample to the same group of countries and sectors. To obtain a more reliable measure of arm's-length trade, we subtract intra-firm trade from export values. Data on GDP (in current USD) and distance are from the CEPII database.

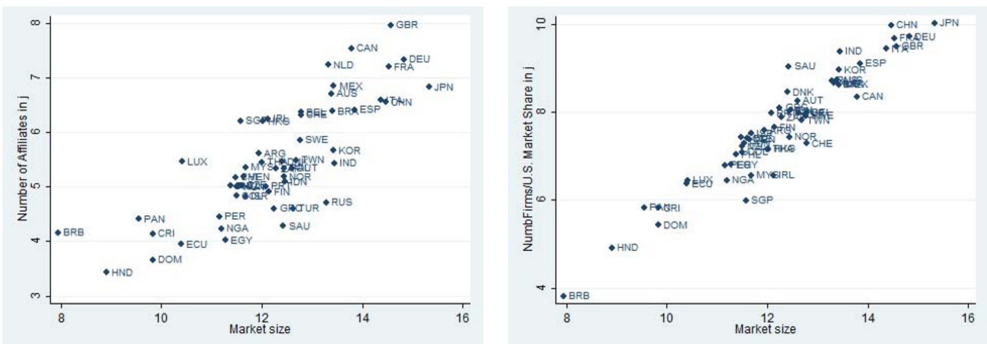
[20]http://www.bea.gov/international/index.htm.

Table 6: List of Countries

Argentina	Greece	Philippines
Australia	Guatemala	Poland
Austria	Hong Kong	Portugal
Bahamas	Honduras	Russian Federation
Barbados	Hungary	Saudi Arabia
Belgium	India	South Africa
Bermuda	Indonesia	Singapore
Brazil	Ireland	Spain
Canada	Israel	Sweden
Chile	Italy	Switzerland
China	Jamaica	Taiwan
Colombia	Japan	Thailand
Costa Rica	Korea, Republic of	Trinidad and Tobago
Czech Republic	Luxembourg	Turkey
Germany	Mexico	
Denmark	Malaysia	
Dominican Republic	Nigeria	
Ecuador	Netherlands	
Egypt	Norway	
Finland	New Zealand	
France	Panama	
United Kingdom	Peru	

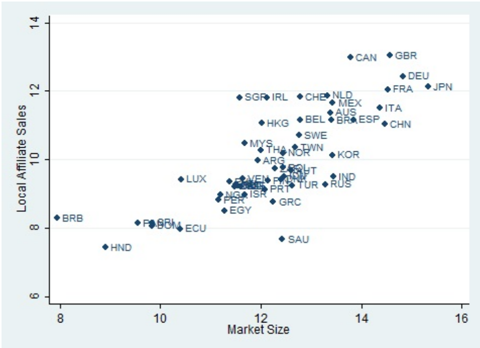
B Facts

Figure 2: Number of U.S. Affiliates in 2004



(a) Entry and Market Size (b) Normalized entry and Market Size

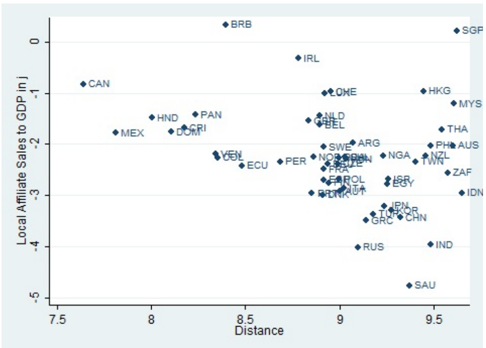
Figure 3: Affiliate Sales and Export in 2004



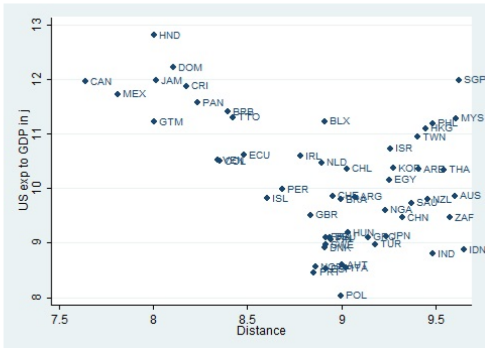
(a) Affiliate Sales and Market Size

(b) Export and Market Size

Figure 4: Affiliate Sales and Export in 2004

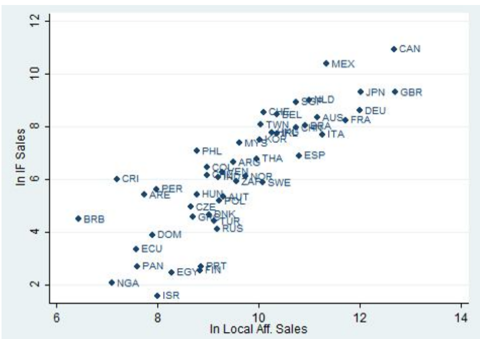


(a) Affiliate Sales and Distance

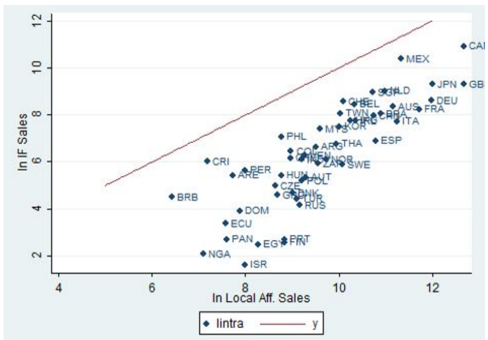


(b) Export and Distance

Figure 5: Intra-firm trade and Local Affiliate Sales in 2004



(a)



(b)

Source: Authors' calculations based on BEA dataset for 2004.

C Profits

In what follows we determine the dividend per share in the economy. In order to do this we use the total profits from exporting from i to j (including also trade within a country):

$$\begin{aligned}\Pi_{ij}^{X} &= w_i L_i \left[\int \frac{1}{\sigma} x_{ij} dG(a) - \int w_j f_{ij}^{X} dG(a) \right] \\ &= \frac{X_{ij}}{\sigma} - w_j f_{ij}^{X} w_i L_i \int dG(a). \end{aligned} \quad (43)$$

Note that when $i = j$, this expression represents domestic profit.[21] Since $n_{ij} = w_i L_i \int_{a_{ij}^{M}}^{a_{ij}} dG(a)$, the expression above can be rewritten as

$$\Pi_{ij}^{X} = \frac{X_{ij}}{\sigma} - n_{ij} w_j f_{ij}^{X}. \quad (44)$$

The total profits for country j's affiliates are:

$$\begin{aligned}\Pi_{ij}^{M} &= w_i L_i \int \frac{1}{\sigma} x_{ij}^{M} dG(a) - \int w_j f_{ji}^{M} dG(a) \\ &= \frac{X_{ij}^{M}}{\sigma} - n^{M} w_j f_{ji}^{M}, \end{aligned} \quad (45)$$

since $n^{M} = w_i L_i \int\limits_{0}^{a_{ij}^{M}} dG(a)$.

Total profits in this economy are

$$\begin{aligned}\Pi &= \sum_i \sum_j \left(\Pi_{ij}^{X} + \Pi_{ij}^{M} \right) \\ &= \sum_i \sum_j \left[\left(\frac{X_{ij}}{\sigma} + \frac{X_{ij}^{M}}{\sigma} \right) - \left(n_{ij} w_j f_{ij}^{X} + n^{M} w_j f_{ij}^{M} \right) \right]. \end{aligned} \quad (46)$$

[21] If we are interested in the domestic profits from serving market i we should compute: $\Pi_{ii} = w_i L_i \int_0^{a_{ii}} \frac{1}{\sigma} x_{ii} dG(a) - \int_0^{a_{ii}} f_{ii} dG(a)$. We should proceed in the same way for computing the number of firms entering a particular market i: $N_{ii} = w_i L_i \int_0^{a_{ii}} dG(a)$. This expression delivers the overall number of firms existing in i.

This expression is the sum of the overall profits produced by domestic, exporting and multinational firms in every country. Remember that country j is receiving varieties from $N-1$. More specifically, total sales in country j are determined by varieties sold by domestic firms, varieties exported to j, and varieties produced locally by foreign affiliates. Hence, total import in country j are $\sum_i \left(X_{ij}^X + X_{ij}^M\right) = Y_j$, where we used the fact that trade is balanced. Substituting the equilibrium number of exporters and affiliates we can rewrite the worldwide profits as:

$$\Pi = \sum_j \left[\frac{Y_j}{\sigma} - \lambda_4^{-b} Y_j\right] = Y \frac{1-\lambda_4^{-b}}{\sigma}. \tag{47}$$

Hence dividends per share are:

$$\begin{aligned}\pi &= \frac{\Pi}{\sum_i w_i L_i} = \frac{\Pi}{Y}(1+\pi) = \frac{1-\lambda_4^{-b}}{\sigma}(1+\pi) \\ &= \frac{\frac{1-\lambda_4^{-b}\sigma}{\sigma}}{\left(1-\frac{1-\lambda_4^{-b}\sigma}{\sigma}\right)}. \end{aligned} \tag{48}$$

D Price Index

The price index is

$$\begin{aligned} P_j^{1-\sigma} &= (\sigma/(\sigma-1))^{1-\sigma}[k/(k-\sigma+1)] \\ &\times \sum_{k=1}^{N} w_k L_k \left[(a_{kj}^M)^{k-\sigma+1}\left[\left(w_j^{1-\eta}\left(w_k \tau_{kj}\right)^{\eta}\right)^{1-\sigma} - \left(w_k \tau_{kj}\right)^{1-\sigma}\right]\right. \\ &+ \left. a_{kj}^{k-\sigma+1}\left(w_k \tau_{kj}\right)^{1-\sigma}\right]. \end{aligned} \tag{49}$$

Plugging the productivity thresholds from (18) and (19) we can solve for the price index in the destination country j,

$$\begin{aligned} P_j^{1-\sigma} &= (\sigma/(\sigma-1))^{1-\sigma}[k/(k-\sigma+1)]\sum_{k=1}^{N} w_k L_k \\ &\times \left\{ \left[\lambda_1 \frac{w_j f_{kj}^M - w_j f_{kj}^X}{Y_j} \frac{P_j^{1-\sigma}}{\left(w_j^{1-\eta}\left(w_k\tau_{kj}\right)^{\eta}\right)^{1-\sigma} - \left(w_k\tau_{kj}\right)^{1-\sigma}} \right]^{1-b} \right. \\ &\times \left[\left(w_j^{1-\eta}\left(w_k\tau_{kj}\right)^{\eta}\right)^{(1-\sigma)} - \left(w_k\tau_{kj}\right)^{1-\sigma} \right] \\ &+ \left. \left[\lambda_1 \frac{w_j f_{kj}^X}{Y_j} \frac{P_j^{1-\sigma}}{\left(w_k\tau_{kj}\right)^{1-\sigma}} \right]^{1-b} \left(w_k\tau_{kj}\right)^{1-\sigma} \right\}, \end{aligned} \tag{50}$$

where $b = k/(\sigma-1)$, w_k is the wage paid to workers in country k for firms which are exporting the good, while w_j is the wage paid to the workers who are either producing the j-domestic varieties or the intermediate good y_2 used by the foreign affiliate in country j. Then solving for $P_j^{1-\sigma}$

$$\begin{aligned} P_j^{b(1-\sigma)} &= (\sigma/(\sigma-1))^{1-\sigma}[k/(k-\sigma+1)]\lambda_1^{1-b}\left(Y_j\right)^{b-1} \\ &\times \sum_{k=1}^{N} w_k L_k \left[\left(w_j f_{kj}^M - w_j f_{kj}^X\right)^{1-b} \left[\left(w_j^{1-\eta}\right)^{1-\sigma} w_k{}^{\eta(1-\sigma)}\phi_{kj}^{\eta} - \left(w_k\right)^{1-\sigma}\phi_{kj} \right]^b \right. \\ &+ \left. \left[w_j f_{kj}^X\right]^{1-b} \left(\left(w_k\right)^{1-\sigma}\phi_{kj} \right)^b \right], \end{aligned}$$

where $\phi_{kj} = \tau_{kj}^{1-\sigma}$.

$$\begin{aligned} P_j &= \left[(\sigma/(\sigma-1))^{1-\sigma}\left(k/(k-\sigma+1)\right)\lambda_1^{1-b} \right]^{\frac{1}{b(1-\sigma)}} \left(Y_j\right)^{\frac{b-1}{b(1-\sigma)}} \\ &\times \left[\sum_{k=1}^{N} \frac{Y_K}{Y}\frac{Y}{1+\pi} \left[\left(w_j f_{kj}^M - w_j f_{kj}^X\right)^{1-b} \left[\left(w_j^{1-\eta}\right)^{1-\sigma} w_k{}^{\eta(1-\sigma)}\phi_{kj}^{\eta} - \left(w_k\right)^{1-\sigma}\phi_{kj} \right]^b \right. \right. \\ &+ \left. \left. \left[w_j f_{kj}^X\right]^{1-b} \left(\left(w_k\right)^{1-\sigma}\phi_{kj} \right)^b \right] \right]^{\frac{1}{b(1-\sigma)}}, \end{aligned}$$

which after rearrangements becomes:

$$P_j = \lambda_2 Y_j^{\frac{b-1}{b(1-\sigma)}} \theta_j \left(\frac{Y}{1+\pi} \right)^{\frac{1}{b(1-\sigma)}}. \tag{51}$$

E Proofs

In what follows we provide proofs of the propositions and equilibrium variables.

E.1 Proposition 1

Proof. Total exports from i to j are given by:

$$X_{ij}^{X} = w_i L_i \int_{\overline{a}_{ij}^{M}}^{\overline{a}_{ij}} x_{ij}^{X} dG(a). \tag{52}$$

A firm will be exporting if $a(v) \leq \overline{a}_{ij}$. Using (24), (25), (22) and (23) and the specific assumption about the distribution of the labor unit requirement, a, we obtain:

$$X_{ij}^{X} = w_i L_i \int_{\overline{a}_{ij}^{M}}^{\overline{a}_{ij}} \lambda_3 \theta_j^{\sigma-1} \left(\frac{Y_j}{Y}\right)^{\frac{1}{b}} (1+\pi)^{\frac{1}{b}} (w_i \tau_{ij})^{1-\sigma} a^{1-\sigma} dG(a), \tag{53}$$

$$\text{with } \overline{a}_{ij}^{1-\sigma} = \lambda_4 \frac{w_j f_{ij}^{X}}{(w_i \tau_{ij})^{1-\sigma}} \theta_j^{1-\sigma} \left(\frac{Y}{Y_j}\right)^{\frac{1}{b}} (1+\pi)^{-\frac{1}{b}}, \tag{54}$$

$$\text{and } (\overline{a}_{ij}^{M})^{1-\sigma} = \lambda_4 \frac{w_j f_{ij}^{M} - w_j f_{ij}^{X}}{\left(w_j^{1-\eta} (w_i \tau_{kj})^{\eta}\right)^{1-\sigma} - (w_i \tau_{ij})^{1-\sigma}} \theta_j^{1-\sigma} \left(\frac{Y}{Y_j}\right)^{\frac{1}{b}} (1+\pi)^{-\frac{1}{b}}. \tag{55}$$

Using the assumption of the Pareto distribution and the productivity thresholds, we can then solve the integral and find (28). ■

E.2 Proposition 2

Proof. Total affiliate sale in country j are given by:

$$X_{ij}^{M} = w_i L_i \int_{0}^{\overline{a}_{ij}^{M}} x_{ij}^{M} dG(a). \tag{56}$$

A firm will open a subsidiary in country j if $a(v) \leq \overline{a}_{ij}^M$. Using (25) and (19) and the specific assumption about the distribution of the labor unit requirement, a, we obtain:

$$X_{ij}^M = w_i L_i \int_0^{a_{ij}^M} \lambda_3 \theta_j^{\sigma-1} \left(\frac{Y_j}{Y}\right)^{\frac{1}{b}} (1+\pi)^{\frac{1}{b}} \left(w_j^{1-\eta} \left(w_i \tau_{ij}\right)^{\eta}\right)^{(1-\sigma)} a^{1-\sigma} dG(a), \tag{57}$$

$$\text{with } (\overline{a}_{ij}^M)^{1-\sigma} = \lambda_4 \frac{w_j f_{ij}^M - w_j f_{ij}^X}{\left(w_j^{1-\eta} \left(w_i \tau_{kj}\right)^{\eta}\right)^{1-\sigma} - \left(w_i \tau_{ij}\right)^{1-\sigma}} \theta_j^{1-\sigma} \left(\frac{Y}{Y_j}\right)^{\frac{1}{b}} (1+\pi)^{-\frac{1}{b}}. \tag{58}$$

Then solving the integral we get (29). ■

Notice that if both the intermediates are produced at home, $\eta = 1$, there will be no firm supplying via MP because the cost will be prohibitive (trade costs plus greater fixed cost, $f_{ij}^M > f_{ij}^X$). Thus every firm will end up being an exporter, since it is more profitable. In this case the only gravity equation will be for export sales, as in Chaney (2008):

$$X_{ij}^X = \beta \frac{Y_i Y_j}{Y} \theta_j^{b(\sigma-1)} (f_{ij}^X)^{1-b} \left(w_i \tau_{ij}\right)^{-k}. \tag{59}$$

On the other side, when all the intermediates are produced in the foreign location, $\eta = 0$, we are back in the Helpman et al. (2004) framework. In this scenario, the gravity equations for export and affiliate sales are:

$$X_{ij}^X = \beta \frac{Y_i Y_j}{Y} \theta_j^{b(\sigma-1)} \left(w_i \tau_{ij}\right)^{1-\sigma} \left[\left(\frac{f_{ij}^X}{\left(w_i \tau_{ij}\right)^{1-\sigma}}\right)^{1-b} - \left(\frac{f_{ij}^M - f_{ij}^X}{w_j^{1-\sigma} - \left(w_i \tau_{ij}\right)^{1-\sigma}}\right)^{1-b}\right], \tag{60}$$

$$X_{ij}^M = \beta \frac{Y_i Y_j}{Y} \theta_j^{b(\sigma-1)} w_j^{1-\sigma} \left(\frac{f_{ij}^M - f_{ij}^X}{w_j^{1-\sigma} - \left(w_i \tau_{ij}\right)^{1-\sigma}}\right)^{1-b}. \tag{61}$$

In this Helpman et al. (2004) set up there is no role for complementarity between trade and MP.

F Intensive and Extensive Margin Elasticities

In what follows we derive in details the intensive and extensive margins for affiliate and export sales.

F.1 Intensive and Extensive Margins of Affiliate Sales

1) Rearranging the definition of intensive and extensive margins of affiliate sales we get

$$-\frac{\partial X_{ij}^M}{\partial \tau_{ij}}\frac{\tau_{ij}}{X_{ij}^M} = \underbrace{-\frac{\tau_{ij}}{X_{ij}^M}\left(w_i L_i \int_0^{\overline{a}_{ij}^M} \frac{\partial x_{ij}^M}{\partial \tau_{ij}} dG(a)\right)}_{\text{Intensive Margin Elasticity}} \underbrace{-\frac{\tau_{ij}}{X_{ij}^M}\left(w_i L_i x_{ij}^M G'\left(\overline{a}_{ij}^M\right)\frac{\partial \overline{a}_{ij}^M}{\partial \tau_{ij}}\right)}_{\text{Extensive Margin Elasticity}}. \tag{62}$$

Using the definition of equilibrium individual affiliate sales, (25), and assuming that country i is small enough so that $\partial \theta_j^{\sigma-1}/\partial \tau_{ij} \approx 0$, we get:

$$\begin{aligned}
\frac{\partial x_{ij}^M}{\partial \tau_{ij}} &= \eta\left(1-\sigma\right)\tau_{ij}^{\eta(1-\sigma)-1}\left(w_j^{1-\eta}\left(w_i d_{ij}\right)^{\eta}\right)^{1-\sigma}\lambda_3 \theta_j^{\sigma-1}\left(\frac{Y_j}{Y}\right)^{\frac{1}{b}}\left(1+\pi\right)^{\frac{1}{b}} a^{1-\sigma} \\
&= \eta\left(1-\sigma\right)\frac{x_{ij}^M}{\tau_{ij}}.
\end{aligned} \tag{63}$$

Therefore, the elasticity of the intensive margin of affiliate sales with respect to the variable costs is:

$$\begin{aligned}
\varepsilon_{I,\tau_{ij}}^M &= -\frac{\tau_{ij}}{X_{ij}^M}\left(w_i L_i \int_0^{\overline{a}_{ij}^M} \frac{\partial x_{ij}^M}{\partial \tau_{ij}} dG(a)\right) \\
&= -\eta\left(1-\sigma\right)\frac{\tau_{ij}}{X_{ij}^M}\frac{w_i L_i \int_0^{\overline{a}_{ij}^M} x_{ij}^M dG(a)}{\tau_{ij}} \\
&= \eta\left(\sigma-1\right).
\end{aligned} \tag{64}$$

2) Using the definition of the equilibrium productivity threshold from

(23), we find:

$$\begin{aligned}\frac{\partial \overline{a}_{ij}^{M}}{\partial \tau_{ij}} &= -\overline{a}_{ij}^{M}\frac{\left(\eta\frac{\left(w_j^{1-\eta}(w_i\tau_{ij})^{\eta}\right)^{1-\sigma}}{\tau_{ij}}-\frac{(w_i\tau_{ij})^{1-\sigma}}{\tau_{ij}}\right)}{\left(\left(w_j^{1-\eta}\left(w_i\tau_{ij}\right)^{\eta}\right)^{1-\sigma}-\left(w_i\tau_{ij}\right)^{1-\sigma}\right)} \\ &= -\frac{\overline{a}_{ij}^{M}}{\tau_{ij}}\underbrace{\frac{\left(\eta\left(w_j^{1-\eta}\left(w_i\tau_{ij}\right)^{\eta}\right)^{1-\sigma}-\left(w_i\tau_{ij}\right)^{1-\sigma}\right)}{\left(\left(w_j^{1-\eta}\left(w_i\tau_{ij}\right)^{\eta}\right)^{1-\sigma}-\left(w_i\tau_{ij}\right)^{1-\sigma}\right)}}_{=\Gamma}. \qquad (65)\end{aligned}$$

The sign of this derivative is ambiguous. It is positive for low level of τ, but than when τ increases it becomes negative. If the elasticity of substitution is high, the ambiguity is preserved only if τ or/and η are sufficiently low.

We now rewrite the equation for firm level affiliate sales in (25), as

$$x_{ij}^{M} = \lambda_{ij}^{M} a^{1-\sigma}. \qquad (66)$$

Then, since the Pareto distribution assumption implies that $G'(a) = ka^{k-1}$, the aggregate affiliate sales equation becomes:

$$\begin{aligned}X_{ij}^{M} &= w_i L_i \int_0^{\overline{a}_{ij}^{M}} x_{ij}^{M} dG(a) \\ &= w_i L_i \int_0^{\overline{a}_{ij}^{M}} \lambda_{ij}^{M} a^{1-\sigma} k a^{k-1} da \\ &= w_i L_i \lambda_{ij}^{M} (\overline{a}_{ij}^{M})^{1-\sigma} (\overline{a}_{ij}^{M})^{k} \left(k/(k-\sigma+1)\right) \\ &= w_i L_i x_{ij}^{M} G'\left(\overline{a}_{ij}^{M}\right) \frac{\overline{a}_{ij}^{M}}{k-\sigma+1}, \qquad (67)\end{aligned}$$

where we used the fact that $\overline{a}_{ij}^{M} G'\left(\overline{a}_{ij}^{M}\right) = k(\overline{a}_{ij}^{M})^{k}$. Using equation

(67), we can find a solution for the elasticity of the extensive margin:

$$\begin{aligned}
\varepsilon^M_{E,\tau_{ij}} &= -\frac{\tau_{ij}}{X^M_{ij}}\left(w_i L_i x^M_{ij} G'\left(\overline{a}^M_{ij}\right)\frac{\partial \overline{a}^M_{ij}}{\partial \tau_{ij}}\right) \\
&= -\frac{\tau_{ij}}{X^M_{ij}} w_i L_i x^M_{ij} G'\left(\overline{a}^M_{ij}\right)\left(-\overline{a}^M_{ij}\frac{\eta\left(\frac{\left(w_j^{1-\eta}(w_i\tau_{ij})^{\eta}\right)^{1-\sigma}}{\tau_{ij}} - \frac{(w_i\tau_{ij})^{1-\sigma}}{\tau_{ij}}\right)}{\left(w_j^{1-\eta}\left(w_i\tau_{ij}\right)^{\eta}\right)^{1-\sigma} - \left(w_i\tau_{ij}\right)^{1-\sigma})}\right) \\
&= -\frac{\tau_{ij}}{X^M_{ij}}\frac{X^M_{ij}}{\tau_{ij}}(k-\sigma+1)\frac{-\left(\eta\left(w_j^{1-\eta}\left(w_i\tau_{ij}\right)^{\eta}\right)^{1-\sigma} - \left(w_i\tau_{ij}\right)^{1-\sigma}\right)}{\left(w_j^{1-\eta}\left(w_i\tau_{ij}\right)^{\eta}\right)^{1-\sigma} - \left(w_i\tau_{ij}\right)^{1-\sigma})} \\
&= (k-\sigma+1)\frac{\left(\eta\left(w_j^{1-\eta}\left(w_i\tau_{ij}\right)^{\eta}\right)^{1-\sigma} - \left(w_i\tau_{ij}\right)^{1-\sigma}\right)}{\left(w_j^{1-\eta}\left(w_i\tau_{ij}\right)^{\eta}\right)^{1-\sigma} - \left(w_i\tau_{ij}\right)^{1-\sigma})}. \qquad (68)
\end{aligned}$$

F.2 Intensive and Extensive Margins of Export Sales

1) Rearranging the definition of intensive and extensive margins of exports we get

$$-\frac{\partial X^X_{ij}}{\partial \tau_{ij}}\frac{\tau_{ij}}{X^X_{ij}} = \underbrace{-\frac{\tau_{ij}}{X^X_{ij}}\left(w_i L_i \int_{\overline{a}^M_{ij}}^{\overline{a}_{ij}} \frac{\partial x^X_{ij}}{\partial \tau_{ij}} dG(a)\right)}_{\text{Intensive Margin Elasticity}} \underbrace{-\frac{\tau_{ij}}{X^X_{ij}} w_i L_i \left[x^X_{ij} G'\left(\overline{a}_{ij}\right)\frac{\partial \overline{a}_{ij}}{\partial \tau_{ij}} - x^M_{ij} G'\left(\overline{a}^M_{ij}\right)\frac{\partial \overline{a}^M_{ij}}{\partial \tau_{ij}}\right]}_{\text{Extensive Margin Elasticity}}. \qquad (69)$$

Using the definition of equilibrium individual affiliate sales, (24), and assuming that country i is small enough so that $\partial \theta_j^{\sigma-1}/\partial \tau_{ij} \approx 0$, we get:

$$\begin{aligned}
\frac{\partial x^X_{ij}}{\partial \tau_{ij}} &= (1-\sigma)\,\tau_{ij}^{-\sigma}(w_i)^{1-\sigma}\lambda_3\theta_j^{\sigma-1}\left(\frac{Y_j}{Y}\right)^{\frac{1}{b}}(1+\pi)^{\frac{1}{b}} a^{1-\sigma} \\
&= (1-\sigma)\frac{x^X_{ij}}{\tau_{ij}}. \qquad (70)
\end{aligned}$$

Therefore, the elasticity of the intensive margin of export with respect to the variable costs is:

$$\begin{aligned}
\varepsilon^X_{I,\tau_{ij}} &= -\frac{\tau_{ij}}{X^X_{ij}}\left(w_i L_i \int_{\overline{a}^M_{ij}}^{\overline{a}_{ij}} \frac{\partial x^X_{ij}}{\partial \tau_{ij}} dG(a)\right) \\
&= -(1-\sigma)\frac{\tau_{ij}}{X^M_{ij}} \frac{w_i L_i \int_{\overline{a}^M_{ij}}^{\overline{a}_{ij}} x^X_{ij} dG(a)}{\tau_{ij}} \\
&= (\sigma - 1), \qquad (71)
\end{aligned}$$

which is identical to the elasticity in Chaney (2008).

2) In order to derive the extensive margin of trade we need to use the equilibrium productivity thresholds from (22)and (23). Deriving these thresholds with respect to τ_{ij} we find:

$$\frac{\partial \overline{a}^M_{ij}}{\partial \tau_{ij}} = -\overline{a}^M_{ij}\Gamma/\tau_{ij} \qquad (72)$$

$$\frac{\partial \overline{a}_{ij}}{\partial \tau_{ij}} = -\frac{\overline{a}_{ij}}{\tau_{ij}}. \qquad (73)$$

Rewriting the equation for firm level exports in (24), as

$$x^X_{ij} = \lambda^X_{ij} a^{1-\sigma} \qquad (74)$$

allows us to find a connection between the λ^M_{ij} in firm affiliate sales, (66), and λ^X_{ij} in export sales, (74). This implies that firm level affiliate sales can be rewritten as,

$$x^M_{ij} = \underbrace{\lambda^X_{ij} \frac{\left(w_j^{1-\eta}\right)^{1-\sigma}}{\left((w_i \tau_{ij})^{1-\eta}\right)^{1-\sigma}}}_{=\lambda^M_{ij}} (\overline{a}^M_{ij})^{1-\sigma}. \qquad (75)$$

Then since the Pareto distribution assumption implies that $G'(\overline{a}_{ij}) =$

$k(\overline{a}_{ij})^{k-1}$, we can rewrite the aggregate export sales in the following way:

$$\begin{aligned} X_{ij}^X &= w_i L_i \int_{\overline{a}_{ij}^M}^{\overline{a}_{ij}} x_{ij}^X dG(a) \\ &= w_i L_i \int_{\overline{a}_{ij}^M}^{\overline{a}_{ij}} \lambda_{ij}^X a^{1-\sigma} k a^{k-1} da \\ &= w_i L_i \lambda_{ij}^X \left(k/(k-\sigma+1)\right) \left[\lambda_{ij}^X \overline{a}_{ij}^{1-\sigma} - \lambda_{ij}^X (\overline{a}_{ij}^M)^{1-\sigma} (\overline{a}_{ij}^M)^k\right]. \quad (76) \end{aligned}$$

Using the relationship between λ_{ij}^X and λ_{ij}^M highlighted in equation (75), we can modify part of the equation above as

$$\lambda_{ij}^X (\overline{a}_{ij}^M)^{1-\sigma} = x_{ij}^M \left[\left((w_i \tau_{ij})^{1-\eta}\right)^{1-\sigma} / \left(w_j^{1-\eta}\right)^{1-\sigma}\right]$$

which gives

$$\begin{aligned} X_{ij}^X &= w_i L_i \left(1/(k-\sigma+1)\right) \\ &\quad \times \left[x_{ij}^X G'\left(\overline{a}_{ij}\right) \overline{a}_{ij} - x_{ij}^M \underbrace{\left[\left((w_i \tau_{ij})^{1-\eta}\right)^{1-\sigma} / \left(w_j^{1-\eta}\right)^{1-\sigma}\right]}_{=\omega} G'\left(\overline{a}_{ij}^M\right) \overline{a}_{ij}^M \right] \\ &= w_i L_i \left(1/(k-\sigma+1)\right) x_{ij}^X G'\left(\overline{a}_{ij}\right) \overline{a}_{ij} \\ &\quad - w_i L_i \left(\omega/(k-\sigma+1)\right) x_{ij}^M G'\left(\overline{a}_{ij}^M\right) \overline{a}_{ij}^M. \quad (77) \end{aligned}$$

From equation (77) we can find the solution for the elasticity of the extensive margin of export:

$$\begin{aligned} \varepsilon_{E,\tau_{ij}}^X &= -\frac{\tau_{ij}}{X_{ij}^X} w_i L_i \left[x_{ij}^X G'\left(\overline{a}_{ij}\right) \frac{\partial \overline{a}_{ij}}{\partial \tau_{ij}} - x_{ij}^M G'\left(\overline{a}_{ij}^M\right) \frac{\partial \overline{a}_{ij}^M}{\partial \tau_{ij}} \right] \\ &= -\frac{\tau_{ij}}{X_{ij}^X} w_i L_i \left[x_{ij}^X G'\left(\overline{a}_{ij}\right) \left(-\frac{\overline{a}_{ij}}{\tau_{ij}}\right) - x_{ij}^M G'\left(\overline{a}_{ij}^M\right) \frac{\Gamma}{\tau_{ij}} \right]. (78) \end{aligned}$$

Rewriting equation (67) to get:

$$w_i L_i x_{ij}^M G'\left(\overline{a}_{ij}^M\right)\overline{a}_{ij}^M = (k-\sigma+1)\,X_{ij}^M, \tag{79}$$

and equation (77) to obtain:

$$\begin{aligned} w_i L_i x_{ij}^X G'\left(\overline{a}_{ij}\right)\overline{a}_{ij} &= (k-\sigma+1)\left[X_{ij}^X + w_i L_i \left(\omega/(k-\sigma+1\right) x_{ij}^M G'\left(\overline{a}_{ij}^M\right)\overline{a}_{ij}^M\right] \\ &= (k-\sigma+1)\left[X_{ij}^X + \omega X_{ij}^M\right]. \end{aligned} \tag{80}$$

The expressions in (79) and (80) can now be plugged in equation (78), to find a more compact expression for $\varepsilon_{E,\tau_{ij}}^X$. This yields:

$$\begin{aligned} \varepsilon_{E,\tau_{ij}}^X &= -\frac{\tau_{ij}}{X_{ij}^X}\left[(k-\sigma+1)\left[X_{ij}^X+\omega X_{ij}^M\right]\left(-\frac{1}{\tau_{ij}}\right) - (k-\sigma+1)\,X_{ij}^M\left(-\frac{\Gamma}{\tau_{ij}}\right)\right] \\ &= -\frac{\tau_{ij}}{X_{ij}^X}(k-\sigma+1)\frac{1}{\tau_{ij}}\left[-\left(X_{ij}^X+\omega X_{ij}^M\right)+X_{ij}^M\Gamma\right] \\ &= -\frac{1}{X_{ij}^X}(k-\sigma+1)\left[X_{ij}^M(\Gamma-\omega)-X_{ij}^X\right] \\ &= -(k-\sigma+1)\left[\frac{X_{ij}^M}{X_{ij}^X}(\Gamma-\omega)-1\right], \end{aligned} \tag{81}$$

where

$$\Gamma = \frac{\left(\eta\frac{\left(w_j^{1-\eta}(w_i\tau_{ij})^\eta\right)^{1-\sigma}}{\tau_{ij}} - \frac{(w_i\tau_{ij})^{1-\sigma}}{\tau_{ij}}\right)}{\left(w_j^{1-\eta}\left(w_i\tau_{ij}\right)^\eta\right)^{1-\sigma} - \left(w_i\tau_{ij}\right)^{1-\sigma}} \tag{82}$$

$$\omega = \left[\left(\left(w_i\tau_{ij}\right)^{1-\eta}\right)^{1-\sigma} / \left(w_j^{1-\eta}\right)^{1-\sigma}\right], \tag{83}$$

Notice that $\Gamma > \omega$ is true for certain parameter restrictions consistent with our calibration exercise.

We can conclude that

$$\text{if } X_{ij}^M > X_{ij}^X \longrightarrow \varepsilon_{E,\tau_{ij}}^X < 0, \tag{84}$$

$$\text{if } X_{ij}^M < X_{ij}^X \longrightarrow \varepsilon_{E,\tau_{ij}}^X > 0. \tag{85}$$

G Welfare

In this section, we show how to derive equations (36)-(38). From the profit of the threshold exporting firm we retrieve the value of export

$$\frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{(1-\sigma)} Y_j \left(w_i a_{ij} \tau_{ij}\right)^{1-\sigma} / P_j^{1-\sigma} = w_j f_{ij}^X. \tag{86}$$

Rearranging (86) we get

$$\left(\frac{\sigma}{\sigma-1} w_i \tau_{ij}\right)^{(1-\sigma)} = \frac{w_j f_{ij}^X \sigma P_j^{1-\sigma}}{a_{ij}^{1-\sigma} Y_j}. \tag{87}$$

Total export sales are

$$\begin{aligned} p_{ij} q_{ij} &= \frac{Y_j p_{ij}}{P_j^{1-\sigma}} \\ &= Y_j a^{1-\sigma} \left(\frac{\sigma}{\sigma-1} w_i \tau_{ij}\right)^{(1-\sigma)} P_j^{\sigma-1} \\ &= \left(\frac{a}{a_{ij}}\right)^{1-\sigma} w_j \sigma f_{ij}^X. \end{aligned} \tag{88}$$

From this last equation, we can compute average export sales:

$$\begin{aligned} (\overline{p_{ij} q_{ij}})^X &= \int_{a_{ij}^M}^{a_{ij}} \left(\frac{a}{a_{ij}}\right)^{1-\sigma} w_j \sigma f_{ij}^X \frac{g(a)}{G(a_{ij}) - G(a_{ij}^M)} da \\ &= \int_{a_{ij}^M}^{a_{ij}} \frac{a^{k-\sigma}}{a_{ij}^{1-\sigma}[a_{ij}^k - (a_{ij}^M)^k]} w_j \sigma f_{ij}^X da \\ &= \left(\frac{k\sigma}{k-\sigma+1}\right) \left(\frac{a_{ij}^{k-\sigma+1} - (a_{ij}^M)^{(k-\sigma+1)}}{a_{ij}^{1-\sigma}[a_{ij}^k - (a_{ij}^M)^k]}\right) w_j f_{ij}^X. \end{aligned} \tag{89}$$

Following a similar procedure we can obtain average affiliate

$$
\begin{aligned}
(\overline{p_{ij}q_{ij}})^M &= \int_0^{a_{ij}^M} \left(\frac{a}{a_{ij}^M}\right)^{1-\sigma} w_j \sigma f_{ij}^M \frac{g(a)}{G(a_{ij}^M)} da \\
&= \left(\frac{k\sigma}{k-\sigma+1}\right) w_j f_{ij}^M, \qquad (90)
\end{aligned}
$$

and domestic sales

$$
\begin{aligned}
(\overline{p_{jj}q_{jj}})^D &= \int_0^{a_{jj}} \left(\frac{a}{a_{jj}}\right)^{1-\sigma} w_j \sigma f_{jj} \frac{g(a)}{G(a_{jj})} da \\
&= \left(\frac{k\sigma}{k-\sigma+1}\right) w_j f_{jj}. \qquad (91)
\end{aligned}
$$

We analyze the behavior of the average export sales, and show that: (i) the ratio including the difference in the cutoffs is lower than 1; and, (ii) average sales decline with a reduction in average productivity of exporters, while it increases with a reduction in average productivity of multinationals.

Proposition 5

$$
\left(\frac{a_{ij}^{k-\sigma+1} - (a_{ij}^M)^{(k-\sigma+1)}}{a_{ij}^{1-\sigma}[a_{ij}^k - (a_{ij}^M)^k]}\right) < 1
$$

i.e., average export sales are smaller than in models with only exporting firms.

Proof.

$$
\begin{aligned}
a_{ij}^{k-\sigma+1} - (a_{ij}^M)^{(k-\sigma+1)} &< a_{ij}^{1-\sigma}[a_{ij}^k - (a_{ij}^M)^k] \\
a_{ij}^{k-\sigma+1} - (a_{ij}^M)^{(k-\sigma+1)} &< a_{ij}^{k-\sigma+1} - a_{ij}^{1-\sigma}(a_{ij}^M)^k \\
(a_{ij}^M)^{(k-\sigma+1)} &> a_{ij}^{1-\sigma}(a_{ij}^M)^k \\
(a_{ij}^M)^{(1-\sigma)} &> a_{ij}^{1-\sigma}
\end{aligned} \qquad (92)
$$

which is true since $a^M < a_{ij}$ and $\sigma > 1$. ■

Proposition 6 *Average export sales are decreasing (increasing) in average productivity of exporters (multinationals).*

Proof. We start by taking the derivative

$$\frac{\partial(\overline{p_{ij}q_{ij}})^X}{\partial a_{ij}} = \frac{(k-\sigma+1)(a_{ij}^M)^{k-\sigma}\left(a_{ij}^{1-\sigma}[a_{ij}^k-(a_{ij}^M)^k]\right)-\left(a_{ij}^{k-\sigma+1}-(a_{ij}^M)^{k-\sigma+1}\right)\left[(k-\sigma+1)a_{ij}^{k-\sigma}-(1-\sigma)a_{ij}^{-\sigma}(a_{ij}^M)^k\right]}{\left(a_{ij}^{1-\sigma}\left[a_{ij}^k-(a_{ij}^M)^k\right]\right)^2}.$$

Consider the sign of the numerator:

$$\begin{aligned}
(k-\sigma+1)(a_{ij}^M)^{k-\sigma}\left(a_{ij}^{1-\sigma}[a_{ij}^k-(a_{ij}^M)^k]\right) &< \left(a_{ij}^{k-\sigma+1}-(a_{ij}^M)^{k-\sigma+1}\right) \\
&\quad \times\left[(k-\sigma+1)a_{ij}^{k-\sigma}-(1-\sigma)a_{ij}^{-\sigma}(a_{ij}^M)^k\right] \\
\frac{(a_{ij}^M)^{k-\sigma+1}-a_{ij}^{1-\sigma}(a_{ij}^M)^k}{a_{ij}^{k-\sigma+1}-(a_{ij}^M)^{k-\sigma+1}} &< 1+\frac{(\sigma-1)a_{ij}^{\sigma}(a_{ij}^M)^k}{a_{ij}^{(k-\sigma+1)(k-\sigma)}} \\
\frac{1-(a_{ij}^M/a_{ij})^k}{1-(a_{ij}^M/a_{ij})^{k-\sigma+1}} &< 1+\frac{\sigma-1}{k-\sigma+1}\frac{(a_{ij}^M)^k}{a_{ij}k} \\
\underbrace{\frac{1}{1-(a_{ij}^M/a_{ij})^{k-\sigma+1}}}_{>0\text{ and }<1}-\underbrace{\frac{(a_{ij}^M/a_{ij})^k}{1-(a_{ij}^M/a_{ij})^{k-\sigma+1}}}_{>0\text{ and }<1} &< 1+\underbrace{\left(\frac{a_{ij}^M}{a_{ij}}\right)^k}_{<1}\underbrace{\frac{\sigma-1}{k-\sigma+1}}_{<1}.
\end{aligned}$$

As

$$\frac{1}{1-(a_{ij}^M/a_{ij})^{k-\sigma+1}} < \frac{(a_{ij}^M/a_{ij})^k}{1-(a_{ij}^M/a_{ij})^{k-\sigma+1}},$$

we can conclude that $\partial(\overline{p_{ij}q_{ij}})^X/\partial a_{ij} < 0$. ■

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