UMR 8184
Thema
THéorie Économique, Modélisation et Applications

Thema Working Paper n°2013-10
Université de Cergy Pontoise, France

FRENCH ECONOMISTS AND THE PURCHASING POWER OF MONEY

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2012

UNIVERSITÉ de Cergy-Pontoise
CNRS CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

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When French economists read *The Purchasing Power of Money*, they were primarily interested in the equation of exchange and the reformulation that Fisher proposed regarding the quantity theory of money. This reading led them to ponder the meaning that should be given to this theory and to study its empirical significance. Some of them, namely Rueff and Divisia, went further still and considered Fisher's work as a starting point for their own analyses, which were related in particular to the monetary index, the integration of money into general equilibrium theory and the analysis of monetary phenomena in an open economy.

Keywords: quantity theory of money, price index, theory of purchasing power parity, marginal utility of money, integration of money into general equilibrium.
JEL Classification: B 10, E 40, F 31

◊ Une version antérieure de ce texte a été présenté au colloque international à l'occasion du centenaire de l'ouvrage Le pouvoir d'achat de la monnaie d'Irving Fisher, 14 et 15 octobre 2011 à Lyon

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As soon as the *Purchasing Power of Money*[1] was published, French economists emphasised the importance of the book. They were not short on praise. "Here," wrote Simiand (1912: 704-5), "is a masterly work… Through its efforts of precision and rigour… demonstrations, through the clarity of the analyses… this book seems to offer of the quantity theory … a presentation that we are within our bounds to declare as the most exact, complete, well-reasoned and critical analysis currently in existence." His merit was that he had, by formulating the equation of exchange, changed the existing conception of quantity theory and the argumentation in support of it.

However, it was this very point of merit that provoked criticism from certain quarters, since, in the early 20th century, the quantity theory of money was, in France, the subject of great controversy: its theoretical foundations and empirical pertinence were called into question. The idea whereby "one of the normal effects of an increase in the quantity of money is an exactly proportional increase in the general level of prices[2]" (Fisher 1911: 157) was rejected by Lescure, Nogaro, Aftalion and Simiand. They dismissed it as it seemed to them to be empirically ill-founded. They did not limit themselves to underlining, as De Foville did (1907: 145), that variations in prices are not proportional to variations in the quantity of money because the velocity of circulation of the money cannot be considered constant and because paper — banknotes, but also bills of exchange — can substitute metallic currencies as a means of payment. They called into question the meaning of the causality. In the relationship that was established between money and prices, they refused to acknowledge that "it is the variations in monetary factors independent of prices that lead the game and affect prices" (Aftalion, 1925: 672). They explained that, on the contrary, it is the variations in prices that, in large part, command the movements of monetary factors. In times of prosperity, the increase in prices provokes a demand for credit that increases the amount of deposits and the velocity of circulation. In recessions, the fall in prices and stagnation of activity flood the banks with money that has become useless, diminishing the amount of deposits and the velocity of circulation. To support their argument, they often evoked the relation that Pierre des Essarts (1895) had highlighted between the economic fluctuations and the velocity of

[1] A French translation of Fisher's book was published much later, in 1926, precisely at a time when Divisia's and Rueff's contributions revived the debate; but French economists did not wait for the publication of this translation to read and discuss the *Purchasing Power of Money*.

[2] Fisher, when he formulated this proposition, made reference to Aupetit. In fact, the latter was more cautious. Theoretical analysis led him to claim that, in the case of metallic currency, the relationship between the quantity of money and its value depends on the elasticity of the demand for the money commodity in relation to the price (Aupetit, 1901: 132). He concludes his empirical study (*Ibid.:* 281) by affirming that an increase in the monetary supply leads, all else being equal, to an increase in prices. He does not, then, invoke the idea of proportionality.

circulation. In short, they reject the idea that the variations in the quantity of money are *the cause* of the variations in prices. But they do not limit themselves to opposing facts to the quantity theory, they criticised Fisher for suggesting a purely mechanical explanation of the evolution of prices, whereas Political Economy must provide an analysis of economic phenomena that attains the root of their profound causes, that is, the behaviour of individuals, their motives and needs. The problem for these economists was that of knowing whether the criticisms addressed to the simpler version of the quantity theory – the one that held that variations in prices obey variations in the quantity of money and are proportional to them – can also be applied to Fisher's analyses.

While many French economists criticised the quantity theory, others supported it (Denis, 1938). Rist[3] (1914), for example, held that when one studies the evolution of prices and the quantity of money, in France, from 1878 to 1910, one cannot fail to be struck by the parallelism of the two series and that it is difficult to deny any influence from the monetary factor on the increase in prices. They welcomed Fisher's book favourably (Dechesne, 1914), but the interpretation that they made failed to contribute many new elements. Later, in the 1920s, Rueff (1922 and 1927) and Divisia (1925-1926 and 1928) wanted to go further. Distrustful of the statistical works that claimed to falsify the quantity theory, they returned to the question of indices, stressing the importance — if they were to be used to such ends — of deducing them logically from the equation of exchange. Generally, Fisher had based his arguments in *The Purchasing Power of Money* on a closed economy. But after WWI, the question of exchange became, at least in Europe, a key question. Aftalion (1926: 782) fiercely attacked the theory of purchasing power parity that he presented as a logical consequence of the quantity theory. To the sequence whereby, in a regime of flexible exchange rates, the variations in the quantity of money determine the evolution of domestic prices, which themselves determine the variations in exchange rates, he opposed the idea that the variations in exchange rates determine the evolution of prices. Divisia and Rueff rejected this thesis. In order to justify his point of view, Rueff was obliged to reformulate the theory of purchasing power parity by extending to commodities the analysis that defines the import and export gold points (Crouhy-Veyrac and alii, 1982: 327). He thus thought he could transpose the approach that Fisher had adopted for analysing the purchasing power of money to the question of exchange. Although Fisher (1911: 174) briefly evokes the manner in which the equation of exchange can be integrated into models of general equilibrium, he does not deal with the

[3] Rist does not make reference in this article to Fisher's book. It is likely, therefore, that he had not, at that time, read it.

problem — this was not his objective. Divisia (1928) was to provide a systematic analysis of this question.

Hence, there were two ways of reading *The Purchasing Power of Money*. One could simply try to understand it in order to interpret and debate it. But it was also possible to look for ideas that would allow for further analysis of monetary phenomena. We will rely on this distinction to explain the way in which French economists read *The Purchasing Power of Money*.

1. *THE REACTIONS TO THE PUBLICATION OF THE PURCHASING POWER OF MONEY*

From this book, French economists retained above all the formulation of the equation of exchange that Fisher provided and the way in which he proposed a reformulation, based on this relation, of the quantity theory of money. It was this aspect of the book that interested them and struck them as innovative. They paid more discreet attention to his analysis of transition and cycles. The propositions that Fisher formulated to stabilise the value of money were discussed but dismissed by everyone, including those who otherwise accepted and extended his ideas.

1.1.The equation of exchange

In order to determine the purchasing power of money, Fisher (1911: 53) formulated the equation of exchange: the total price of commodities sold equals the total value of the money that was given in exchange. By noting M the quantity of money — notes and coins — V its velocity of circulation, M' the total of call deposits, V' their velocity of circulation, p_i the price of the good i, q_i the quantity that was exchanged, the equation is written as follows:

$$MV + M'V' = \sum_i p_i q_i$$

This equation "constitutes a 'truism'[4] that is so obvious that it seems not to require any further demonstration" (Aftalion, 1911: 410). But what is not obvious is Fisher's choice of approach. In order to analyse the value of money, is it better to reason in terms of the supply and demand of money or to use the equation of exchange? The curious thing is that this key question would not be explicitly posed by French economists until much later[5], despite the

[4] Aftalion repeats the term used by Fisher (1911: 157).

[5] Rist (1938: 372), for example, criticised Fisher for virtually leaving out the analysis of demand by restricting himself to representing it via the overall amount of transactions to be performed, "to such an extent that the theory of the value of money, instead of being regularly inserted in the general theory of the price of commodities – in which the demand element plays such a major role – seems to constitute a separate theory that is in a certain sense 'out of the scope'". In the same way, Baudin (1926), repeating the criticism that Keynes

fact that it implicitly underpins the debates. However, Walras (1874-77: 522) had discussed it at length by way of the modifications that he successively introduced into the various editions of his *Éléments d'économie pure.* The theorists contented themselves with evoking the role played by paper credit in the settlement of certain transactions, but Fisher (1911: 370) had shown, in advance, how these could be integrated into the reasoning. Aftalion (1911: 412) seems, however, to have had doubts. What was missing, he wrote, from Fisher's analysis, "is an explanation of the law that [he] formulates and an explanation that… has a psychological basis. What we must show is how, when monetary quantities increase, individual incomes are modified and how these modifications can determine individuals to pay higher prices for the same commodities."

What primarily attracted attention was the rewriting that Fisher proposed for the equation of circulation by introducing bank deposits subject to check. One would have thought that the economists, who considered cheques and notes to be the basic representation of metallic currency, would have criticised this reformulation. But this was not the case, and Rist (1938: 366) affirms that "Fisher's formula… is… in perfect agreement with [my] conception which sees in bank credit a simple means of circulating money." He justifies his thesis by affirming that the velocity of circulation of the deposits "must therefore be added to that of the primary money in order to measure its effect on prices." Lescure (1912: 463) thinks, on the contrary, that "Fisher is wrong… to introduce quantities of money from heterogeneous factors (fiduciary money, velocity of circulation) into the calculation, because, whatever [Fisher] believes, the quantity of fiduciary money in circulation is independent of the quantities of metallic currency." Whereas Rist concurred, like Fisher, that between *M* and *M'* a normal relationship exists, Lescure held that the quantity of fiduciary money in circulation depends on the size of the commercial transactions and not on the metallic supplies.

1.2.Fisher and the quantity theory of money

For Fisher (1911: 157), "the equation of exchange… is the means of demonstrating… the quantity theory". But what must therefore be understood by the quantity theory? Aftalion (1925: 669) believes that we may suggest two interpretations. The first holds that the variations in prices obey and are more or less proportional to the relationship of the quantity of money to the volume of transactions. A variation in the quantity of money in the narrow

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(1911) addressed to Fisher, reproaches him "for having accurately shown how variations in the monetary supply affect the level of prices, but for not having indicated how this influence occurs."

sense of the term would lead to a proportional variation of deposits and would leave the velocities of circulation and the volume of the transactions unchanged. It was this interpretation of the *Purchasing Power of Money* that Rist (1938: 366) presented and approved. In the second form (Aftalion, 1925: 670), the value of money is still inversely proportionate to its quantity. Yet all the monetary factors, not just *M* but *M', V* and *V'*, must be understood by the quantity of money. It is thus supposed that the demand deposits do not remain in a constant relationship with the money in the narrow sense, and that their augmentation lowers the value of money. Similarly, it is supposed that the velocities of circulation are not constant and that their variations affect the prices. What characterises this second version of the quantity theory is the idea that "the price level is the effect and cannot be the cause of change in other factors" (Fisher, 1911: 182). The key element of the quantity theory lies in the causal action that it lends to monetary factors. Clearly, the same arguments cannot be opposed to both versions of the quantity theory.

A great deal of empirical work was carried out in order to analyse the relationships between monetary factors and prices. The conclusions that may be drawn from them are tenuous, given the brevity of the periods studied (1920-5) and the mediocre quality of available data[6]. The period is characterised by a very great instability. From April 1920 to February 1922 prices dropped by 48% only to rise by 52% between February 1922 and December 1923 (Aftalion, 1925: 658). Evidently, the variations in the quantity of notes are too low to explain the movement of the prices, as the graph — figure 1 — drawn by Rueff (1927: 166) indicates.

[6] At that time, no data on bank deposits were available. Only some establishments published monthly statements of accounts (Rueff, 1927: 105). The most often used price index was the wholesale price index, which introduces a bias into the reasoning as it is undoubtedly too sensitive to variations in international prices and exchange rates.

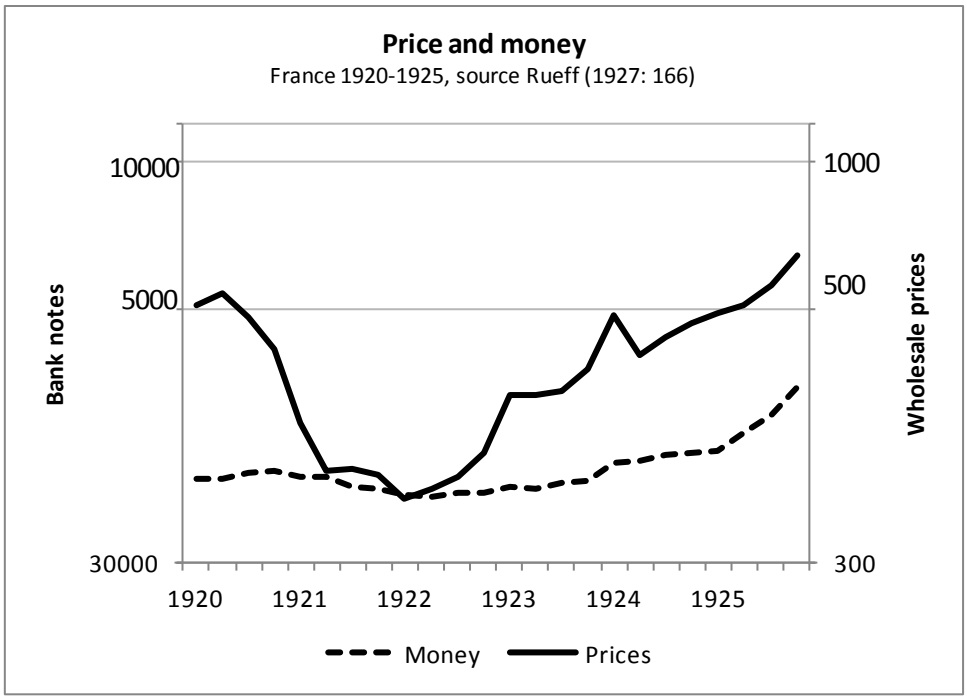

Figure 1

Based on the figures provided by the few banks that published their statements, Rueff (1927: 113) maintains that the deposits vary markedly, as do the notes, but are however affected by a regular and continuous increase. He thus forms an index of the velocity of circulation of bank deposits by relating the amount of bills presented at the clearing house to the amount of deposits. The graphic shows a relationship between the evolution in prices and the velocity of circulation.

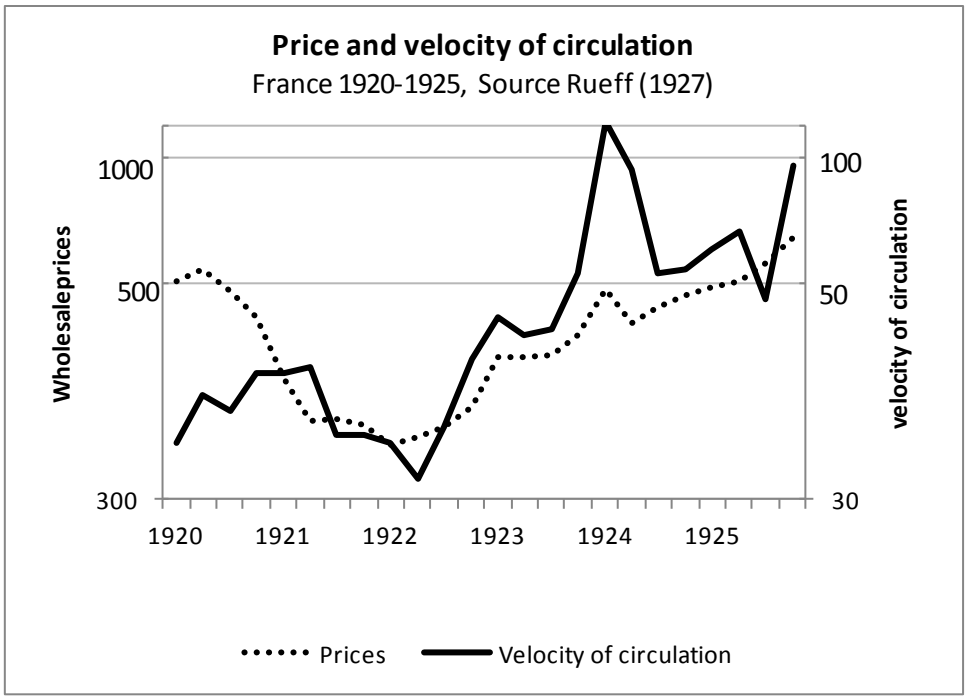

Figure 2 :

The velocity of circulation, as demonstrated by Pierre des Essars (1895) and Fisher (1911: 270), is a cyclical variable: it increases during the expansion phase and decreases during crisis. How can these results be interpreted? Rueff (*Ibid.:* 157) concludes that "it is much more in terms of the variation of velocity of circulation than it is the variation in the amount of monetary supplies that the equation of exchange finds itself satisfied at any given moment." It was to be the second version of quantity theory rather than the first that would be pertinent, but it is possible to maintain the idea that monetary factors determine prices. Aftalion (1925: 673) suggests that we must reverse the order of causality: the progression of prices influences monetary factors, particularly velocities of circulation, rather than obeying them. Clearly, what is missing is an analysis of the factors determining the velocity of circulation of money, or, alternatively, the demand for money.

1.3.The cycles

Fisher (1911: 58-9) holds that a rise in the quantity of gold provokes, during a period of transition, a rise in production: the increase in prices that it involves raises the profits, because the interest rates that the entrepreneurs are paying are not immediately adjusted. When French economists evoke Fisher's theory of the cycle, they ignore this argument: they dismiss it without even discussing it. Lescure and Aftalion support a real theory of the cycle whereby the expansionist effect of a real shock is progressively spread within the economy by

way of the increase in revenue that it provokes. Of course, since in their analyses the increase in production is accompanied with a rise in prices, the monetary factors — defined as $MV+M'V'$ — must also go up, but their increase is the effect of the increase in nominal income. "During phases of prosperity, prices go up. But this is because the values constituting the second term of the equation of exchange are progressing – because both the gold coin in circulation (as is revealed by the decrease in bank receipts) and the fiduciary substitutes for money are progressing, as is the velocity of circulation. During the depression, the fall in prices is the result of the decrease of the circulation of gold coin (which returns to the banks), of a decline in recourse to fiduciary procedures, and of the slowing down of the two velocities of circulation, both metallic and fiduciary." (Aftalion, 1913: 325)

Divisia (1928: 167-8) holds, on the contrary, "that crises are not so much crises of production as they are *monetary* crises and that what creates them has much more to do with *lack of credit* than it does a lack of demand". His analysis of the cycle stresses the role of credit and the prices stickiness. Let us consider an economy in which money is convertible to gold. An increase in credit increases the *circulation*[7], as would an issue of notes, but since certain prices are not instantaneously adjusted, it causes an increase in activity. But this process cannot continue indefinitely, at least not in a country in which the issue of notes is subject to an effective limitation. Credit is therefore more difficult and rare, which has the effect of reducing the discrepancy between transactions and payments. The increase in the volume of transactions in a situation in which the circulation can no longer increase involves a fall in prices that causes the crisis. Divisia's analysis is absolutely typical. It is entirely based on the equation of exchange and on the opposition between adjustment and steady state. In this respect, it evokes that of Fisher, but no reference is made here to interest rates: the whole analysis rests on the prices stickiness and on the limits that convertibility imposes on the development of circulation. In that respect, it moves away from Fisher's analysis.

1.4.Compensated dollar

The propositions that Fisher (1911, 1913) made in order to stabilise the purchasing power of metallic currency raised the interest of French economists but they did not adhere to it. Olbrechts (1914) maintained that the variations in weight of the virtual dollar would only provoke a proportional variation in its purchasing power if they led to a proportional variation in the quantity of money. In his opinion, it was highly unlikely that this was the case.

[7] By this expression, Divisia understands all of the monetary factors, which we may express in Fisher's notation as *MV + M'V'*.

Certainly, as Fisher explained (1911: 343), a rise in the official price of gold would incite the public to buy ingots at the Treasury, which would reduce the monetary mass. But Olbrechts held that, in order for this reduction to be sufficient, the industrial demand for gold must progress in proportions that were not realistic. He thus opposed to Fisher's propositions arguments taken from quantity theory. Fisher (1913: 421) thought the increase in prices at the start of the 20th century was due to the discovery of new gold mines and economical methods of extraction. According to Divisia (1926: 67), this meant falsely attributing variations in prices to their unit of measure, whereas these were the effects of the interaction of a series of factors. The automatic correction that Fisher proposed to instigate would simply introduce an additional reaction, without there being any hope of stabilising the system. Divisia doubted even the very interest of a policy of stabilising the value of money. Any long term contract contains risks associated with variations in prices. But what was important for the contracting parties was not the variation of prices in general, but that of the price of particular products. It is the practice of hedging that allows such a risk to be reduced. The correction via the monetary index proposed by Fisher would only be useful if it were possible to ignore the individual variations in prices in the light of variations in the overall level of prices – for instance, during periods of high inflation. Overall, it was better to maintain commodity money, like the gold standard, whose variations were not considerable in relation to other prices.

2. *THE PURCHASING POWER OF THE MONEY* AS A STARTING POINT

Divisia and Rueff, the French economists who were closest to Fisher's approach, presented in rather different ways the relationship between their work and *The Purchasing Power of Money*. When Rueff (1927: 47) introduced the equation of exchange, in his *Théorie des phénomènes monétaires*, he underlined the fact "that it had been completely brought to light by…Irving Fisher" to whom he paid homage. On the contrary, Divisia (1925 b: 999) wished to preserve the originality of his contribution. He acknowledged, naturally, that his "law of circulation" was no different from the equation of exchange. However, he claimed that, when he discovered it, he had absolutely no knowledge of Fisher's book. In fact, he gave a slightly different presentation of it, seeking to generalise it by introducing a circulatory function C of the various characteristics of money, in place of the product of the quantity of money through its velocity of circulation. The circular law is formulated, by noting P the price index and Q that of the payments, $C = kPQ$ where k is a constant. He specifies the circulatory function by distinguishing the money M, the deposits M' and the advances that the banks accord to their clients M''. He thus writes, by noting α the relationship of the advances

to the deposits, $C = MV + M'(1+\alpha)V'$. The advantage of this formula is that it explains the role of the bank credits in the determination of the value of money. This proposition is "a *law-definition* that asks nothing from experience" (Divisia, 1925 c: 1130). However, if such is the case, Fisher's attempt to experimentally verify his equation of exchange has no analytical meaning.

> "The only conclusion to be drawn is that the indices that he uses are indeed those that must be included in the formula or rather that they give, in the practice, results very nearly theoretical indices; that, furthermore, his other estimations are not false… The circular law has not to be verified; it must serve, on the contrary, to verify the accuracy of observational data or to calculate or study some of his terms in relation to the others. In this respect, the circular law is an *instrument of observation or analysis.*" (*Ibid.*)

Divisia and Rueff undertook the task of pursuing Fisher's work. Their approach testifies to their desire to rigorously articulate economic theory and empirical observation[8]. One essential step in implementing this project was developing the tools necessary for observing reality, and this task was a theoretical task, since in order for a measure to make sense, it must correspond to the theoretical notion that it proposes to evaluate. They thus returned to the question of the definition of the monetary index. Beyond this question, they parted company. While Fisher repeatedly evoked international exchange in his book, he did not provide a systematic treatment of it. Rueff thus undertook to bridge this gap. For his part, Divisia sought to resolve the question of the integration of money into general equilibrium theory, a question to which, in his opinion, mathematician economists — he was thinking more of Pareto than of Walras — had not found a satisfactory answer.

2. 1. The monetary index

In order to study the relationships between the quantity of money and its purchasing power, economists and statisticians were using indices designed for other uses. Aftalion (1925: 657), for instance, used the wholesale price index, which he considered "to be approximately representative of all prices". He gave no precise theoretical and empirical justification in support of this assertion. In fact, he was unable to do so, as he had not defined what a monetary index should be. Naturally, all empirical studies involve approximations but it is still important to know what one intends to measure in order to have an idea of the error that may be made when a given index is used.

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[8] Divisia was a founding member of the *Econometric Society* and its president in 1935. Subsequently he was also president of the *International Econometric Society*.

Fischer, Rueff and Divisia started with the same idea: the monetary index must be deduced from the equation of exchange. Fisher (1911: 364) explained that it must vary proportionally to the quantity of money when the volume of the transactions and the velocity of circulation remain constant. It is like that when the index is the arithmetic mean of the elementary price indices weighted by the value of the goods exchanged during the base period. It is not the only possible solution, but it is one solution. He thus showed that an index of the quantities corresponds to each price index, such that the product of the two indices is equal to the value of the goods exchanged (*Ibid.:* 385).

For Rueff (1927: 61), the price index must remain constant when, for a given supply of products existing on the market, the overall value of the demand remains unchanged in spite of the variation in prices, the fall in one compensating for the increase in the other, where p_i is the price of the commodity *i*, and q_i is its quantity. If, when the price of p_i becomes p_i' the overall value of the demand remains unchanged, the index of the overall level of prices *P* must remain unchanged:

$$\sum p_i q_i = \sum p_i' q_i \qquad \Leftrightarrow \qquad P = P'$$

If the index is a weighted arithmetic mean, the weights must be as follows:

$$\frac{\sum \omega_i p_i}{\sum \omega_i p_{i,0}} = \frac{\sum \omega_i p_i'}{\sum \omega_i p_{i,0}}$$

where $p_{i,0}$ is the price of the commodity *i* during that base period. It will be thus when the weights affected to each price will be the quantities exchanged during the current period, $\omega_i = q_i$. The price index *P* is written:

$$P = \frac{\sum p_i q_i}{\sum p_{i,0} q_i}$$

The index of the quantities, *Q*, is deducted from the price index based on the idea that the product of the two indices must be equal to the value of the goods exchanged during the unity of time, by one constant factor (*Ibid.*: 73).

In order to define the indices of price and quantity, Divisia reasons in continuous time and substitutes his circular law for the equation of exchange. Since, in the transactions in which money intervenes, the value of the goods exchanged is equal to the monetary payments, we may write:

$$C=\sum p_i q_i \tag{1}$$

The prices and the quantities are functions of time. By differentiating (1), we obtain:

$$\frac{dC}{C}=\frac{\sum p_i dq_i}{\sum p_i q_i}+\frac{\sum q_i dp_i}{\sum p_i q_i}$$

Furthermore, the circular equation $C = kPQ$ implies

$$\frac{dC}{C}=\frac{dQ}{Q}+\frac{dP}{P}$$

By writing that the two expressions of variations of the circulation are equal and by observing that, in each equation, one term corresponds to the variation of quantities whereas the other corresponds to the variation of prices, it is possible to conclude that the price and quantity indices can be defined by the relationships:

$$\frac{dP}{P}=\frac{\sum q_i dp_i}{\sum p_i q_i} \qquad \frac{dQ}{Q}=\frac{\sum p_i dq_i}{\sum p_i q_i} \tag{2}$$

In the equations (2) the prices and the quantities are functions $p(t)$ and $q(t)$ of time, which can be written as follows:

$$\frac{dP}{P}=\frac{\sum q_i(t)\,p_i'(t)\,dt}{\sum q(t)p(t)}=F(t)dt$$

In order to obtain the value of the index at the moment *t*, we must simply integrate this equation, but this integration is only possible if we know the variations of prices and quantities in the full interval between the base period and the moment *t*.

We can draw three lessons from this research. The quantities that occur in the indices are not quantities that are produced or consumed, or even exchanged quantities – they are the quantities that intervene in the payments (Divisia, 1925-6, 2: 1001). Since the aim of constructing the monetary index is to estimate the purchasing power of money, we must include here the price of all of the goods purchased, including real estate, securities, labour and other services (Fisher, 1911: 218; Divisia, 1925 b: 984). The price index must be a chain index. This idea had been clearly stated by Fisher (1911: 203), but the merit of its demonstration clearly falls to Divisia (1925 b: 1005).

2.2. The exchange rates

Fisher (1911: 90-6) opposes two monetary systems: metallic circulation in which the exchange rates are fixed if the nations use the same standard, and inconvertible paper money in which the exchange rates are variable. When the money consists of irredeemable paper money, there is no close relationship between the levels of prices in the various countries. However, when the countries use the same standard, the levels of prices of each profoundly affects the others through the intermediary of the classic mechanism of the distribution of precious metals. In a gold standard regime, the prices in a small country depend on those prevailing abroad. The quantity of money is determined by the international prices. However, we must not consider that the prices of the various goods or even the overall level of prices are the same in the various countries, since the distance, the costs of transport and insufficient knowledge of earning opportunities maintain differences between prices in different countries.

After World War I, the question of determining the exchange rate, in a regime in which money is not convertible, was central to the debates. According to the interpretation proposed by Aftalion (1926: 782; 1927: 430) the purchasing power parity theory considers exchange to be the trade of one purchasing power against another. The "normal" exchange rate is equal to the relationship of the purchasing power of monies, which are themselves determined by the quantity of money. He rejects this analysis, which strikes him as overly mechanical. Certainly the purchasing power of foreign money enters into the estimation that an individual makes of the value of a currency. But other elements come into play. Exchange operations are not necessarily related to a commercial transaction; very often their objective is speculative, people buy a currency in the hopes that it will appreciate so well that the present rate depends on the future rate that they are anticipating. To the purchasing power parity theory, which holds that the evolution of the monetary mass determines the variations in prices that determine the exchange rate, he opposes the idea that it is the anticipated variations of the quantity of money that determine the exchange rate, which, itself, determines the prices and often, the quantity of money[9] (*id.*, 1926 b: 968). While Fisher argued that, under a regime of inconvertibility, domestic prices do not depend on foreign prices, Aftalion affirms to the contrary, that an increase in foreign prices expressed in domestic money affects the domestic prices not only because it increases costs, but also because it raises incomes and therefore demand.

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[9] Divisia (1926: 54), analysing the role of anticipations, notes: "if… the speculators believe in the quantitative law, we could see the exchange rate precisely follow the issues and even, in certain cases, precede the issue of notes that is deemed inevitable, which could lead us to believe that the variation of the exchange is the cause of the issue, whereas it would have only been the anticipated consequence of it."

Divisia (1926: 51-58) maintain ideas close to those of Fisher and thus opposes Aftalion. The difficulty, that we come up against when we discuss the validity of the purchasing power parity, is that this term has not been clearly defined. In fact, what is at stake when we analyse external commerce, are the prices of exportation and importation and not the overall level of prices. In order to demonstrate this, it is practical to base the reasoning on the case of two countries, where q_x is the quantity of the commodity x that has been exported, p_x is its price in the producing country and p_x* its price abroad, e the exchange rate and ξ the exportation expenditure. For a commodity to be exported, its foreign price, wrote Fisher (1911: 92), must be at least equal to its domestic price plus the cost of exportation. Divisia reworks this idea by substituting the equality to the inequality:

$$\sum_x q_x p_x + \xi = e \sum_x q_x p_x^*$$

The same applies for the importations:

$$\sum_z q_z p_z - \zeta = e \sum_z q_z p_z^*$$

q_z is the quantity of the commodity z that is imported, p_z is its price in the importing country, p_z* is its price abroad, ζ represents the importation costs. The result of these two relationships is that the exchange rate lies between the relationships of the exported products' domestic price to their price in the foreign country:

$$\frac{\sum_x q_x p_x}{\sum_x q_x p_x^*} < e < \frac{\sum_z q_z p_z}{\sum_z q_z p_z^*}$$

Divisia (1926: 56, note) emphasises that we cannot interpret this inequality as establishing the limits within which the exchange rate is fixed under a forced currency regime: the nature and quantity of imported and exported goods itself depends on the exchange rate. The idea that the exchange rate is determined with respect to the purchasing power of the two monies is unfounded.

The variations in the exchange rate affect the monetary index, because the exchange is one of its constitutive elements. But the effect of the exchange rate on the value of money is more difficult to analyse because its variations would lead to the variation, the increase or decrease, of a large number of prices. The only way to proceed is to study the effects of the

exchange rate on the variables that enter into the circulation equation. It does not seem to be able to sustainably affect the velocity of circulation and the volume of payments. Since, under an irredeemable paper money, the authorities retain control of the quantity of money, the exchange rate has no lasting effect on the value of money.

In the article entitled "*Le change, phénomène naturel*", Rueff (1922: 646) acknowledges that as an initial approximation, "the money of Country (1) is exchanged for that of Country (2) at a rate that will give, inside Country (2), a purchasing power that is roughly equal to that operating within Country (1)". He calls the difference between a country's internal purchasing power and its purchasing power abroad a "disparity" of the money of Country (1) into Country (2). The stability of the equilibrium of the balance of payments [10] is assured by the variations of disparity. Any deficit of the balance of payments causes an increase in the disparity that tends to restore equilibrium. Based on these definitions, he developed an empirical analysis that shows that the disparities are limited and that they correlate to the total balance of payments.

The theoretical arguments that Rueff uses in this article are left wide open to criticism. The reference to a poorly defined notion of the purchasing power of money seems inadequate in a context in which the pertinent prices are those of the goods that are subject to international trade. The empirical conclusions are unconvincing since his estimations of disparities are not really inconsiderable. In the case of France and England, for example, they attained 24% in January 1921.

In *La théorie des phénomènes monétaires*, he systematically reworks these questions by generalising to all commodities the notion of the import and export gold points. Let us consider an article *i* whose price in pounds sterling in England is p_i^*. We write *e*, the number of francs required to buy one pound sterling. The price of this article in France is p_i, Rueff (1927: 185) defines the disparity between the English price in France of this article as the quantity:

$$\frac{p_i - ep_i^*}{P}$$

where P is the overall level of prices in France. ζ are the costs of importing this article: transport costs, brokerage fees, customs duties etc. The purchase of this article in England is

[10] He defines the balance of payments as the difference between the total debt payable abroad and that of the external debt owing. Thus, when the balance of payments of one country is in deficit, the number that translates it is positive.

advantageous if $\zeta < p_i - ep_i^*$. Rueff acknowledges that, if this condition is satisfied, all the quantities of this article consumed in France q_i will be imported from England. The value of these importations, $ep_i^* q_i$, will be included in the liabilities of the balance of payments. Analogously, we could write ξ, the costs imposed by the exportation of this commodity from France to England. Purchasing this article in France is advantageous for an English person if $\xi < ep_i^* - p_i$. If this condition is satisfied, all the quantities consumed in England, q_i*, will be imported from France and the value of the exportations, $q_i^* p_i$, will be included in the assets of the balance of payments.

Rueff represents this phenomenon, as shown in Figure 3, by noting the disparity of English prices in France on the horizontal axis and the total of debts or claims that the operations introduce into the French balance of payments on the vertical axis. All of the values are divided by the general index of French prices, that is, expressed in francs from the base period. In order to simplify the graph, we can assume that the product is strictly homogenous and that the delivery fees for the commodities are uniform and do not depend on their places of origin and destination. As long as the disparity is inferior, in absolute value, to the cost of the international exchanges, there are no exportations, or importations. When the disparity exceeds the cost of the importations, the product is imported. We may wonder what has become of the real value of the importations $\left(q_i e p_i^*\right)/P$ when the disparity increases beyond ζ / P. On the figure drawn by Rueff, it is constant. But the graph is misleading, because the disparity, as Rueff acknowledges (1927: 201), cannot exceed ζ since, if this were not the case, the product's importation to France would be so advantageous that many competitors would attempt to do so.

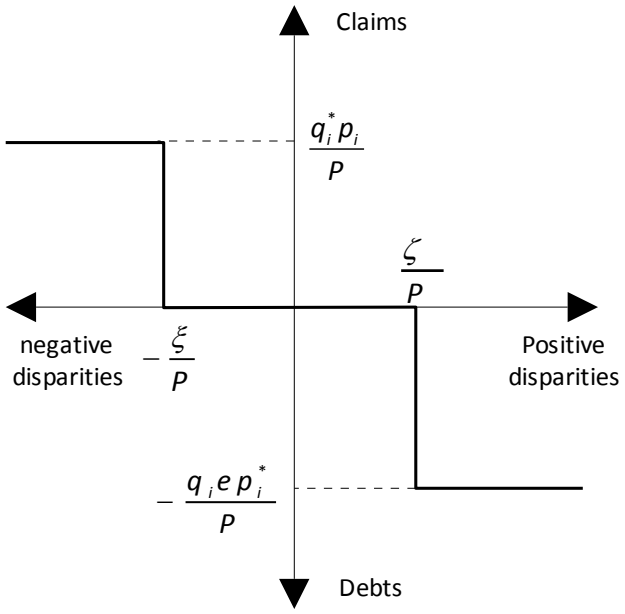

Figure 3 (Rueff, 1927: 189)

If we remove the two simplifying hypotheses — uniformity of the product and transport costs — the diagram will be slightly different, as indicated in Figure 4. When the disparity increases, for a given level of the general index of prices, from $-\xi/P$, the exportations will decrease until they became nil then give way to importations.

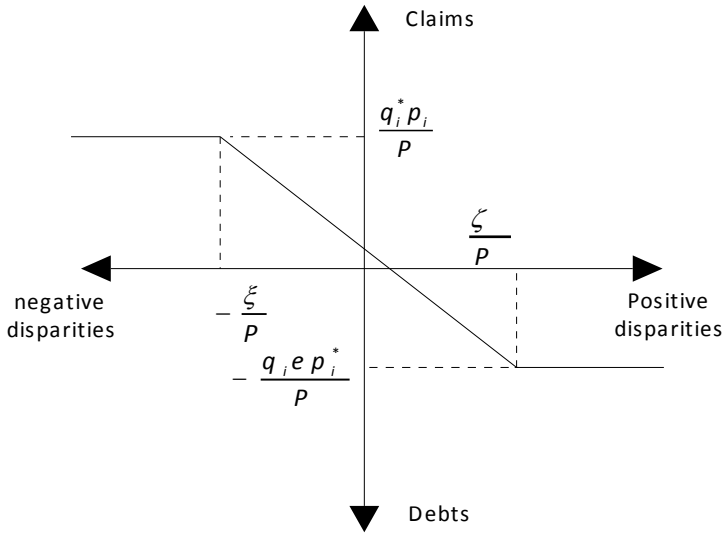

Figure 4 (Rueff, 1927: 198)

Rueff generalised his reasoning by introducing the notion of average disparity. The average disparity between two countries, France and England for instance, is represented by

writing P as the index of the general level of prices in France and P^* as the same index in England:

$$\frac{P - \bar{e}P^*}{P}$$

So that the disparity is nil in the base period, we must introduce into the formula not the exchange rate alone but the exchange rate, $\bar{e}$, related to its coin parity or, in a irredeemable paper money regime, to the exchange rate in the base period. The problem here is the choice of index. While Divisia stressed that it was important to use a price index of external commerce, Rueff worked on the general level of prices, which is hardly satisfactory, but at the time, it was the only series available to him. We can move on from this index of average disparity between two countries to the average disparity of foreign countries by calculating a weighted arithmetic mean. The disparity with each country will be affected by a weighting that is proportional to the variation of the balance of payments that results from the same variation in the partial disparity.

On this basis, Rueff showed that, in the cases studied, the indices remained within the zone of fluctuation that he had defined: the foreign prices evaluated in francs never strayed from the French prices by more than 33%, this figure being the estimation that he made of the transaction costs in international commerce. He concluded that we cannot claim that the internal purchasing power of money is always equal to its foreign purchasing power. The "commodities points theory" (Rueff, 1927: 236) must substitute the purchasing-power-parity theory — a useful but rough approximation. He highlights the fact that, like the equation of exchange, his analysis does not allow a direct causal link to be established between prices and the exchange rate; it only defines "a framework within which phenomena must be placed." (*Ibid.*)

One of the essential themes of Rueff's work is the idea that the variations in prices ensure the stability of the equilibrium[11]. In the case of international exchanges, he claims that everything seems to suggest that "the variations in balance of payments, starting from a position of equilibrium, were provoking variations in average disparity that tended to restore the former equilibrium" (*Ibid.:* 240). In order to demonstrate this assertion, he assumes that, for any commodity, the balance of payments is a decreasing function of the external disparity. Of course, this is not necessarily the case, but he claims that, in the majority of cases, it is

[11] On Rueff's liberalism, see Christopher Chivvis (2010).

possible to adopt a similar diagram of disparity to the one shown in Figure 4. However, this figure clearly shows that, in the case where the domestic production of an article is nil, a depreciation of the domestic money or a drop in the overall level of domestic prices increases the amount of debts payable and the deficit of the balance of payments. The stability of the equilibrium is not self-evident and Rueff does not precisely analyse the conditions that must be satisfied in order for the equilibrium to be stable.

2.3. The integration of money into models of general equilibrium

When Fisher analyses the effects of the quantity of money on his purchasing power, he is stressing three points:

- If the value of money is inversely proportionate to its quantity, this is due to a characteristic that it is the only commodity to possess: it does not have the power to satisfy human needs in any other way than by allowing us to acquire the goods that have this power (Fisher, 1911: 32).
- The opponents of the quantity theory falsely claimed that the level of prices — or in other words, the value of the money — cannot be determined by the equation of exchange because it is, already, by the supply and demand. The economists who support the idea that the supply and demand determine the prices, independently of the quantity of money, its velocity of circulation and the volume of the transactions are forgetting that if we write that the demand of each commodity is equal to its supply, we obtain a system that does not allow us to determine all of the prices, because it contains less independent equations than it does unknown ones. It is the problem of the missing equation (Fisher, 1892: 62) that the equation of exchange compensates for, thus allowing the system to be resolved.
- Fisher (1911: 175) emphasises that we must distinguish between two problems: that of the determination of relative prices and that of the determination of the level of prices. The legitimacy of an approach that separates these two questions appears clearly when we observe that the demand for a commodity not only depends on its price, but on the general level of prices. Individual prices are only fully determined when we introduce the general level of prices into the analysis.

Nogaro (1924: 160) criticised, without explicitly citing them, the economists who, "considering that prices express two categories of relationships, a relationship of exchange between commodities, then a relationship of exchange of commodities with money, are reasoning as though these two categories of relationships were successively established, and *as though the exchanges, once concluded,* provoked a *demand for money*."

Against Nogaro, Divisia bases his argument on two ideas that Fisher had put forward. He repeats the first when he writes: "the ophelimity of money, *considered solely as a means*

of exchange, cannot be defined a priori by the theory of choices and series of indifference." (Divisia, 1928: 400) The quantity of money that an individual possesses is not an argument for its utility function. Certainly, it allows the individual to acquire other goods, but this property does not allow an *indirect* utility function to be defined, of which the money would be an argument. We cannot therefore determine a demand for money as we determine the demand for other goods.

In order to determine monetary prices, Divisia introduces the circulation equation into a model of general equilibrium in which the transactions consist of direct exchanges of commodities. However, he is confronted with a difficulty. Since money is a means of payment, it participates in the budget constraint that, by writing p_i the price of the commodity i, q_{ih} the quantity of the commodity i requested by the individual h, $\bar{q}_{ih}$ the initial endowment of the agent h of the commodity i, $\bar{m}_h$ his initial endowment in money and m_h, his demand of money, is formulated as follows

$$\sum p_i \left(q_{ih} - \bar{q}_{ih} \right) + m_h - \bar{m}_h = 0$$

Thus, there are as many unknown factors, the m_h, as there are agents. However, in the absence of functions of demand for individual money, they cannot be determined. In order to overcome this difficulty, Divisia reasons in a stationary equilibrium. "If we consider *a long period of time,* we can assume (outside of periods of inflation) that the average quantity of money held for each individual… is more or less constant. We shall thus have, by arguing about averages and either about values taken at the given moment » (Divisia, 1928: 411):

$$m_h = \bar{m}_h \qquad \forall h$$

Further, he adds "basically, if we consider a longer period of time, we must assume, as J.-B. Say has rightly pointed out, that 'products are exchanged for products', and that hence any individual that acquires a certain quantity of goods must sell a quantity of other goods, up to an equivalent value." He thus obtains a model[12] that can be separated into two blocks. The equations of the real sector determine the relative prices, the quantities produced and exchanged. The monetary sector, reduced to the equation of exchange, determines the general level of prices. By construction, money is neutral and the prices of goods are proportional to its quantity. We note that the very structure of this model stems from the idea that money does

[12] Patinkin (1956: 203 and 670) attributes the first explicit version of this model to Divisia.

not have its own ophelimity and that we cannot speak of a demand for cash-balance, as Walras did. The model analyses a stationary state that obeys the law of Say in the sense that each agent conserves from one period to the next the same monetary reserve.

CONCLUSION: FROM THE EQUATION OF EXCHANGE TO THE DEMAND FOR CASH-BALANCE

From the time of its publication through to the 1930s, *The Purchasing Power of Money* was the reference work for French economists who interpreted it as the modern, rigorous version of quantity theory. But this theory was hardly popular. It is therefore not surprising that many French economists, while recognising its merits, fiercely criticised it. It was only in the 1920s that Rueff and Divisia, both graduates of the École Polytechnique where they had been students of Clément Colson, used this book to develop their own analyses of monetary phenomena. Here, I have defended the idea that their contributions were certainly original but were nonetheless based on ideas that Fisher had supported.

Progressively, the approach in terms of supply and demand of money became prominent. Jeanneney's reaction (1936: 365) was extreme but, precisely for that reason, typical. He acknowledges that the definition that Fisher proposes for the quantity of money is befitting for a statistical measure. But its usefulness seems very limited to him, because the term *P* that intervenes in the equation represents the monetary index of the transactions and not the value of the money, and because the term *M',* by uniting the credit openings and the deposits, does not allow the causes of variation in the quantity of money to be clearly distinguished. He concludes that Fisher's equation is a "convenient instrument of measure but not a very useful one, and therefore a poor instrument of analysis." He opposes the Cambridge equation to it, insofar as it underlines the importance of individuals' choices between retaining a monetary reserve and exchanging money for goods. Rueff, who emerges as the French economist who most closely matches Fisher's ideas, would later profoundly modify his approach when he published, under the title of *L'ordre Social* [The Social Order], what should have been the second volume of *Phénomènes monétaires* [Monetary Phenomena], devoted to the dynamic. Thenceforth, he reasoned by opposing the effective cash-balance to the demand for cash-balance, the difference between these two terms constituting the causal link between money, demand for goods and prices. The equation of exchange no longer appeared in the analysis.

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