Master Mathematics

The Master in Mathematics offers a high-level two-year training in pure or applied mathematics for careers in research, teaching, insurance and finance.

Introduction

The Master in Mathematics is based on a strong mathematical program with the aim of training high-level specialists in fundamental and applied mathematics. It offers a wide range of job prospects in **academic or industrial research**, **education**, **insurance** and **finance**.

The master addresses a broad variety of topics, all of which are fascinating, and it also offers a great scientific overview to its students. It relies on a recognized research team, the **AGM Research Center in Mathematics** **(UMR CNRS 8088)**, which develops many collaborations with academic and industrial labs.

The Master in Mathematics benefits from a program of **excellence scholarships**, which provides a 7 300 euros annual support to the selected second-year applicants.

Objectives

From the autonomous car to the new generation helicopter, from trading rooms to medical imaging, from communication systems to transport networks, new technologies are at the center of modern societies. Behind each of these advances are new mathematical concepts, which were often discovered during the development of advanced mathematical theories. The Master in Mathematics allows you to discover, understand, and finally apply these concepts.

Admission

Prerequisite

Prerequisites training

The first year is open to candidates holding a bachelor's degree in mathematics or applied mathematics. Getting this diploma with honours will be fully appreciate.

Places

The lectures are mainly given in the Saint-Martin campus.

Internship(s)

Yes, Compulsory

Conditions

- Attending

Information

marie.chef@cyu.fr

+33134256561

The second year is open to candidates holding the first year of a master's degree in mathematics or applied mathematics.

Application

Conditions of applications

Application to the first year of the master

The first year of the Master in Mathematics will supervise 50 students during the 2023-2024 academic year. The application has to be made via the platform Études en France. The application forms consist of:

- the diploma and mark transcripts from the bachelor's degree (in mathematics or applied mathematics),
- the diplomas and mark transcripts for the last four years,
- a motivation letter, which describes in particular the professional project.

Deadline for the application: April 11th, 2023.

Application to the second year of the master

The second year of the Master in Mathematics will supervise 20 students during the 2023-2024 academic year. The application has to be made via the platform Études en France. The application forms consist of:

- the diploma and mark transcripts from the first year of the master's degree (in mathematics or applied mathematics),
- the diplomas and mark transcripts for the last four years,
- a motivation letter, which describes in particular the professional project.

Deadline for the application: April 11th, 2023.

Conditions of specific applications

The application forms for an excellence scholarship can be downloaded at

https://cytech.cyu.fr/lecole/institut-sciences-et-techniques

The completed forms must be submitted to the e-mail addresses

christine.richter@cyu.fr
philippe.gravejat@cyu.fr

Deadline for the application to an excellence scholarship for the second year of the master : April 29th, 2023.

What's next ?

Further studies

The Master in Mathematics can lead to a PhD in pure or applied mathematics.

Program

The first year program aims at discovering and mastering mathematical objects and tools at the heart of the discipline. The second year is a year of specialization towards research, or towards careers in insurance and finance.

First year program

First semester

Calculus of variations, convex analysis and optimization (6 ECTS)

Working group 1 (2 ECTS)

Probability (6 ECTS)

Programming in Matlab, C, C++ (4 ECTS)

Dynamical systems (6 ECTS)

I. Introduction to differential equations
II. Basics of analysis and linear algebra
1. Basics of topology
2. Basics of linear algebra
3. Basics of differential calculus
III. Cauchy–Lipshitz theorem
1. Fixed point theorem and applications
2. Cauchy-Lipschitz theorem
IV. Linear differential equations
1. Equations with constant coefficients
2. Resolvent and perturbation theory
3. Equations with periodic coefficients
V. Nonlinear differential equations
1. Lifetime of solutions
2. Perturbation theory
3. Flows and vector fields
4. Stability analysis

Topology and functional analysis (6 ECTS)
1. Open and closed sets
2. Compact and connected sets
3. Fundamental group
4. Banach spaces
5. Duality theorems
6. Hilbert spaces
7. Bounded operators
8. Unitary operators and projections
9. Spectrum of operators
10. Spectral radius
11. Diagonalization
12. Compact operators

Second semester

Numerical analysis (5 ECTS)

Partial differential equations (5 ECTS)

Working group 2 (2 ECTS)

Programming in R and Python (2 ECTS)

Assistance to internship research (1 ECTS)

Dissertation or internship (5 ECTS)
Two lectures among the next ones:

Algebra (5 ECTS)

Differential geometry (5 ECTS)

I. Advanced differential calculus
1. Basics of differential calculus in $\mathbf{R}^n$
2. Differential calculus in Banach spaces
3. Immersions, submersions, constant rank theorem
II. Submanifolds
1. Equivalent definitions of submanifolds
2. Diffeomorphisms
3. Tangent space and tangent bundle
4. Vector fields and integral curves
III. Surfaces
1. Second fundamental form
2. Elements of Riemannian geometry

Continuous-time stochastic processes (5 ECTS)

Statistics (5 ECTS)

1. Statistical models
2. Basic notions of pointwise estimation: bias, quadratic risk, convergence
3. Construction of estimators: method of moments and maximum likelihood estimation
4. Optimal estimation: sufficient statistics, complete statistics, Lehmann-Scheffé theorem, Fisher information, Cramér-Rao inequality. Exponential families.
5. Confidence regions
6. Statistical tests

Second year program in pure and applied mathematics

Refresher lectures

Algebra and geometry

The goal of this lecture is to review quickly algebraic notions already covered in the bachelor's degree: especially linear algebra and a bit of algebraic structures.

1. Vector spaces
2. Linear maps
3. Eigenvectors and eigenvalues
4. Reduction of endomorphisms
5. Bilinear and quadratic forms
6. Orthogonality
7. Algebraic structures

Analysis and probability

First semester

Distributions and partial differential equations (8 ECTS)

Working group 3 (2 ECTS)

Stochastic processes (8 ECTS)

Dynamical systems (8 ECTS)

One lecture among the next ones:

Finite element methods (4 ECTS)

Modelling (4 ECTS)

This lecture deals with the main themes of numerical modelling. It is aimed at various student profiles: those wishing to deepen their knowledge in numerical analysis, or improve their programming. It will alternate between theoretical sessions (convergence of algorithms,...) and numerical work in Python with Jupyter Lab. The lecture will cover many classical algorithms of numerical analysis, related to mathematical analysis in the broad sense, and will contain precise applications in its exercises. The goal is to obtain strong bases in programming, fluency in coding, and an understanding of the mathematical results at the heart of the algorithms.

1. Graphics
2. Linear systems
a. Direct methods
b. Iterative methods
c. Principal component analysis
3. Numerical integration
a. Newton-Cotes formulas
b. Monte-Carlo method
4. Function approximation
a. Lagrange interpolation
b. Fourier transform
5. Nonlinear systems
a. Iterative methods in dimension one
b. Newton-Raphson algorithm
6. Optimization
a. Least squares
b. Descent algorithms
c. Constraints and Lagrange multipliers
7. Ordinary differential equations
a. Explicit and implicit Euler schemes
b. High order schemes
c. Numerical illustration of the properties of solutions
8. Partial differential equations
a. Introduction to finite differences
b. Poisson equations, transport equations, and heat equations in dimension one

Second semester

Specialization lecture: Analysis (6 ECTS)

Dynamics of parabolic equations

This lecture will deal with the study of evolution equations modelling the phenomena of advection, diffusion and reaction. These partial differential equations appear for example in fluid mechanics, in population dynamics or in combustion. The goal is to understand how the solutions behave asymptotically, either in large time or near the singularities that they could form in finite time. A universal phenomenon will then be observed: resolution into self-similar solutions. The lecture will begin with classical examples of equations for which representation formulas have been discovered, then it will describe the modern analytical framework, which allows to study the solutions in the absence of such formulas (tools of perturbative analysis, harmonic and spectral analysis).

1. Representation formulas for linear equations
2. Local resolution of quasilinear transport equations
3. Classical solutions of the Burgers equation
a. Cauchy problem
b. Self-similar resolution of singularities
4. Heat equation in Lebesgue spaces
a. Semi-group decay estimates
b. Long time behavior
5. Viscous Burgers equation
a. Cauchy problem
b. Convergence towards blow-up profiles or solitary waves
6. Weak solutions of the Burgers equation

a. Evanescent viscosity limit
b. Convergence to blow-up profile
7. Local Cauchy problem for semilinear parabolic equations
8. Formation of singularities for the Keller-Segel system
a. Virial formulas
b. Backward self-similar profiles
9. Stability analysis
a. Renormalization and modulation
b. Spectral theory
c. Nonlinear stability

Graduate school lecture : Regularity for partial differential equations (6 ECTS)

Regularity for partial differential equations: elliptic equations, homogenization and fluid mechanics

The question of whether solutions of Partial Differential Equations (PDEs) are regular or not is central in the field. One of the most famous open problems is that of the global existence of smooth solutions to the Navier-Stokes equations in fluid mechanics, or the finite-time breakdown of regularity (Millenium problem of the Clay's institute). Another famous problem is Hilbert's 19^{th} problem on the regularity of minimizers of certain functionals in the calculus of variations. This problem was solved in three independent works by De Giorgi, Nash and Moser in the late fifties.

The purpose of these lectures is to give some fundamental tools for the analysis of the regularity of PDEs of elliptic or parabolic type. The material presented in the course is well-known to the PDE community since (at least) the eighties. However some results (De Giorgi-Nash-Moser theorem, epsilon-regularity for the Navier-Stokes equation, uniform estimates in homogenization) are still inspiring new mathematical developments today.

Outline of the lecture:
1. Constant or smooth coefficient elliptic equations (Caccioppoli's inequality, perturbative methods)
2. Crash course in Harmonic Analysis (Calderon-Zymund decomposition, analysis of singular integral operators)
3. Regularity for elliptic equations with bounded measurable coefficients (non perturbative methods of Moser and De Giorgi)
4. Improved regularity in homogenization (compactness methods, quantitative approach, Liouville-type theorems)
5. Epsilon-regularity for the Navier-Stokes equations (compactness proof by Lin)

References:
- Evans, Partial Differential Equations.
- Giaquinta and Martinazzi, An Introduction to the Regularity Theory for Elliptic Systems, Harmonic Maps and Minimal Graphs.
- Seregin, Lecture Notes on Regularity Theory for the Navier Stokes Equations.
- Lemarié-Rieusset, The Navier-Stokes Problem in the 21st Century.

Specialization lecture: Geometry and dynamical systems (6 ECTS)

Dissertation or internship (12 ECTS)

Second year program in applied mathematics to finance

Refresher lectures

Algebra and geometry

The goal of this lecture is to review quickly algebraic notions already covered in the bachelor's degree: especially linear algebra and a bit of algebraic structures.

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2. Linear maps
3. Eigenvectors and eigenvalues
4. Reduction of endomorphisms
5. Bilinear and quadratic forms
6. Orthogonality
7. Algebraic structures

Analysis and probability

First semester

Distributions and partial differential equations (8 ECTS)

Modelling (8 ECTS)

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a. Lagrange interpolation
b. Fourier transform
5. Nonlinear systems
a. Iterative methods in dimension one
b. Newton-Raphson algorithm
6. Optimization
a. Least squares
b. Descent algorithms
c. Constraints and Lagrange multipliers
7. Ordinary differential equations
a. Explicit and implicit Euler schemes
b. High order schemes
c. Numerical illustration of the properties of solutions
8. Partial differential equations
a. Introduction to finite differences
b. Poisson equations, transport equations, and heat equations in dimension one

Stochastic processes (8 ECTS)

Three lectures among the next ones:

Statistical learning (4 ECTS)

Financial risk management (4 ECTS)

Time series methods (4 ECTS)

Numerical methods in finance (4 ECTS)

Second semester

Risk measure: Theory and applications (6 ECTS)

Stochastic modelling (6 ECTS)

Dissertation or internship (12 ECTS)